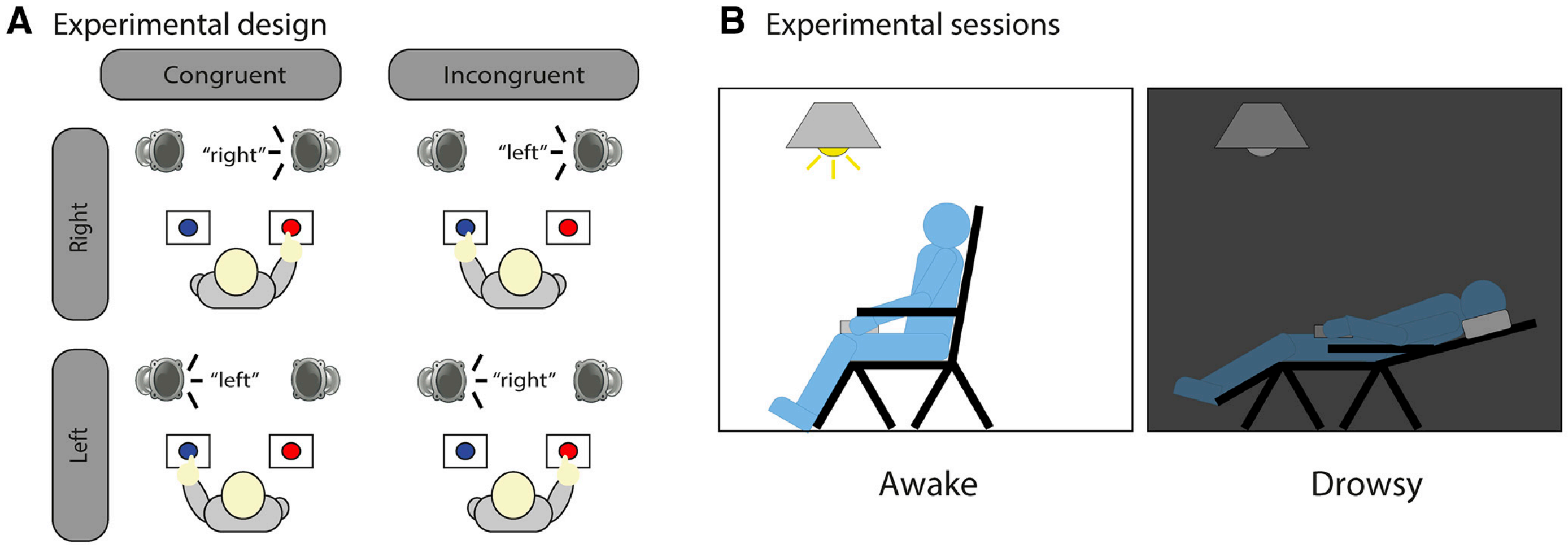
Within-participant information theory + Prevalence Example

Effect of decreased alertness on cognitive control

Behavioural Analysis

- 33 participants did an auditory stroop task (left vs right):
  congruent and **incongruent** trials; **awake** vs **drowsy** experimental conditions
  ~1600 trials per participant



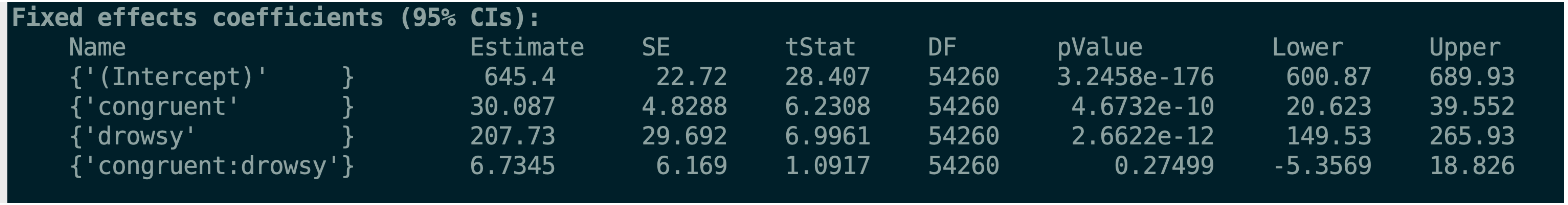
Population mean analysis on Reaction Times

- Linear Mixed Effect Model

```
Linear mixed-effects model fit by ML

Model information:
    Number of observations            54264
    Fixed effects coefficients            4
    Random effects coefficients         132
    Covariance parameters                11

Formula:
    RT ~ 1 + congruent*drowsy + (1 + congruent*drowsy | ID)

Model fit statistics:
    AIC          BIC            LogLikelihood    Deviance
    7.72e+05     7.7213e+05     -3.8598e+05      7.7197e+05

Fixed effects coefficients (95% CIs):
    Name                        Estimate     SE          tStat      DF        pValue          Lower      Upper
    {'(Intercept)'     }         645.4        22.72      28.407     54260     3.2458e-176      600.87     689.93
    {'congruent'       }        30.087       4.8288      6.2308     54260      4.6732e-10      20.623     39.552
    {'drowsy'          }        207.73       29.692      6.9961     54260      2.6622e-12      149.53     265.93
    {'congruent:drowsy'}        6.7345        6.169      1.0917     54260         0.27499     -5.3569     18.826
```

Population mean analysis on Reaction Times

- Linear Mixed Effect Model
- Significant main effect of congruence (30 ms)
- Significant main effect of drowsiness (207 ms)
- No interaction



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Within-participant linear model of mean RT

- We can simply run the same linear model separately within each participant:

```
%% fit within participant models
sub_mdls = cell(1,Nsub);
for subi=1:Nsub
    sub_dat = RT_dat(RT_dat.ID==subi,:);
    mdl = fitlm(sub_dat, 'RT ~ congruent*drowsy');
    sub_mdls{subi} = mdl;
end
```

Within-participant linear model of mean RT

- We can simply run the same linear model separately within each participant:
- 5/33 have a significant main effect of congruence
- 28/33 have a significance main effect of drowsiness
- 0/33 have a significant interaction

Within-participant linear model of mean RT

- What do we learn about the population from this:
- https://estimate.prevalence.online/

Within-participant linear model of mean RT

Congruence

- Evidence to reject the “global null” - ie we can have some confidence the effect exists
- But doesn’t seem to be particularly widespread
- Perhaps experiment underpowered to detect this within participants?

Bayesian prevalence of a statistical test

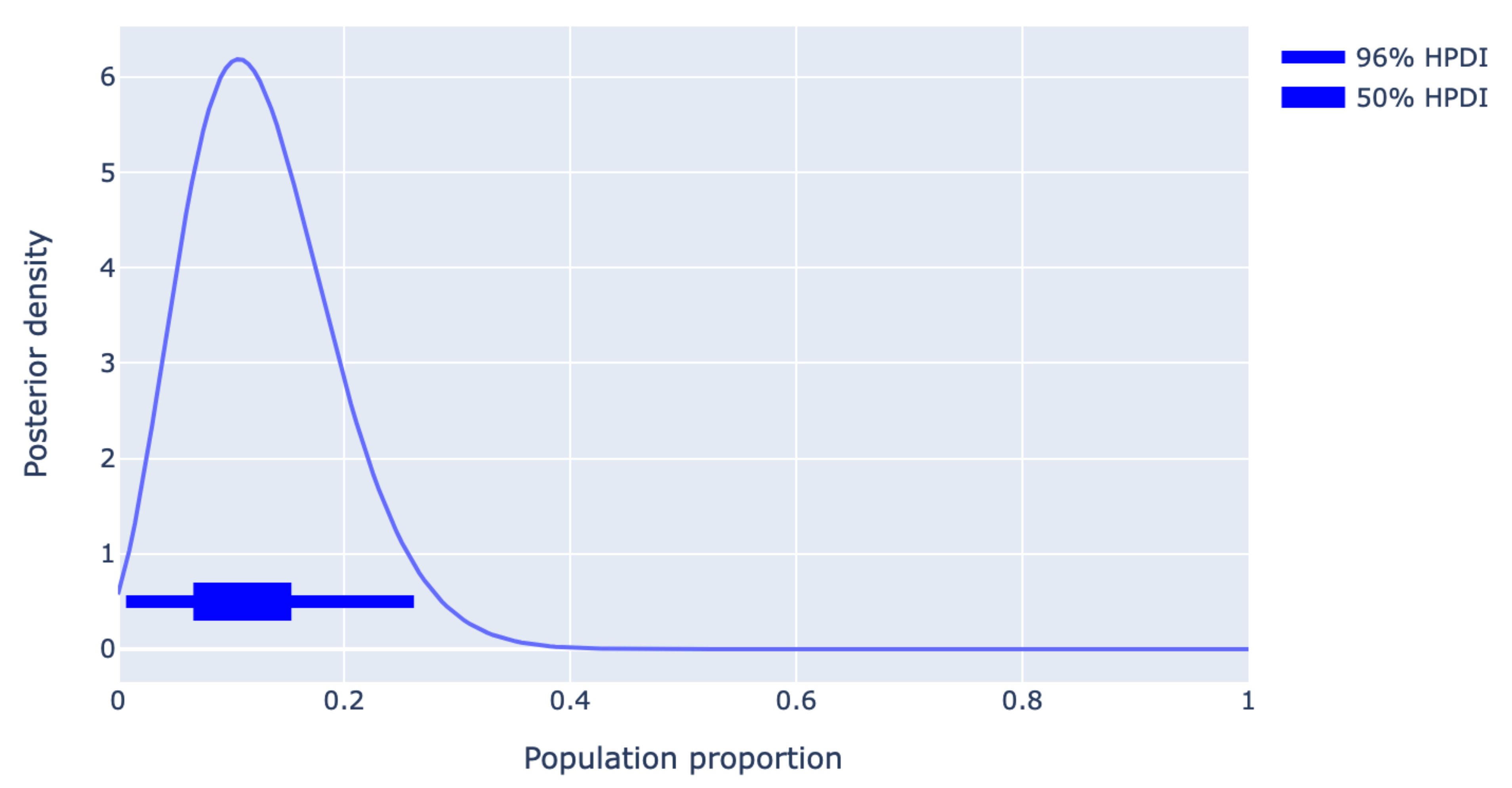
5 out of 33 tests significant with false positive rate 0.05

Bayesian results

Prevalence MAP: 0.11

96% HPDI: [0.01 0.26]

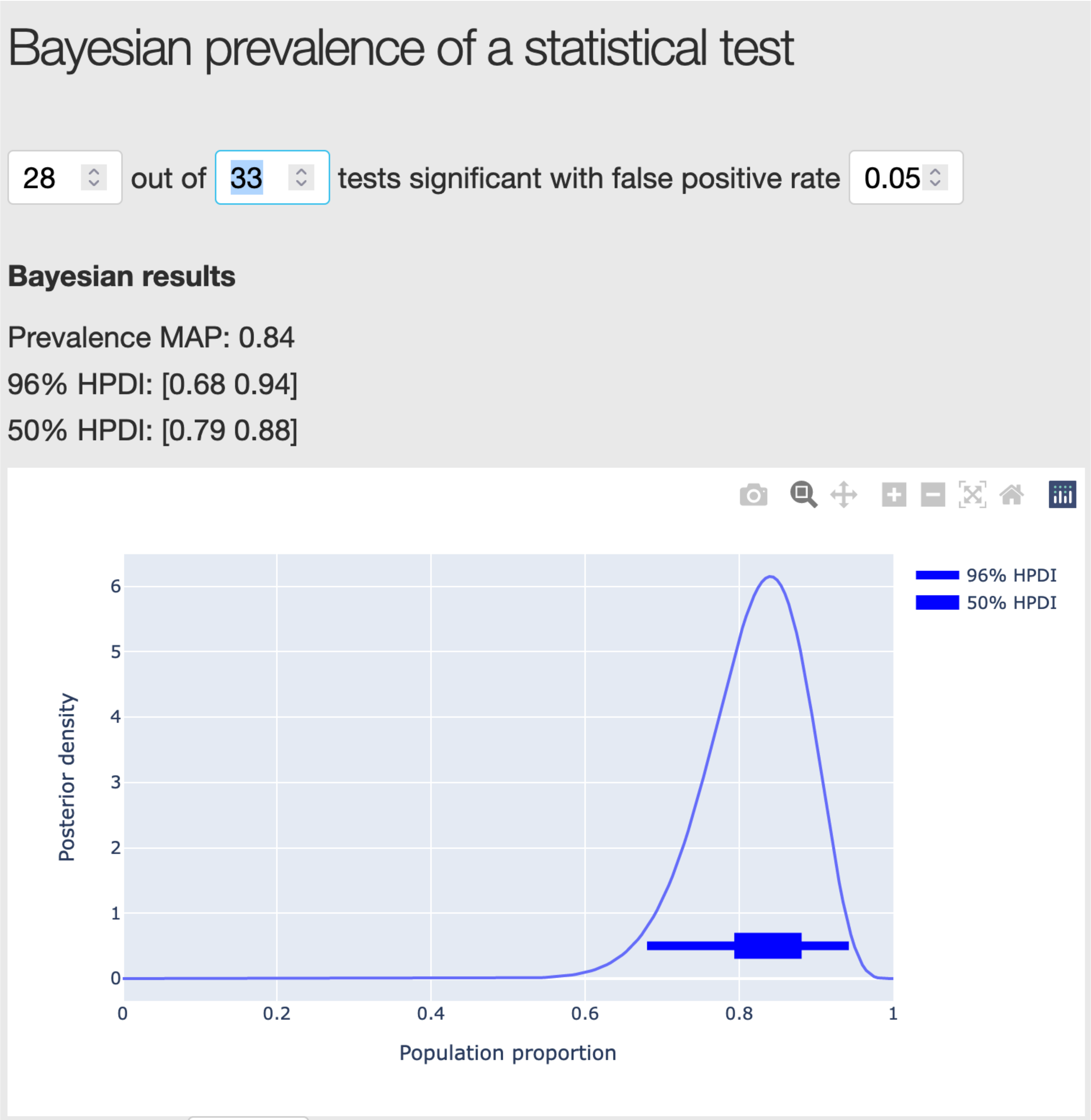
50% HPDI: [0.07 0.15]



Within-participant linear model of mean RT

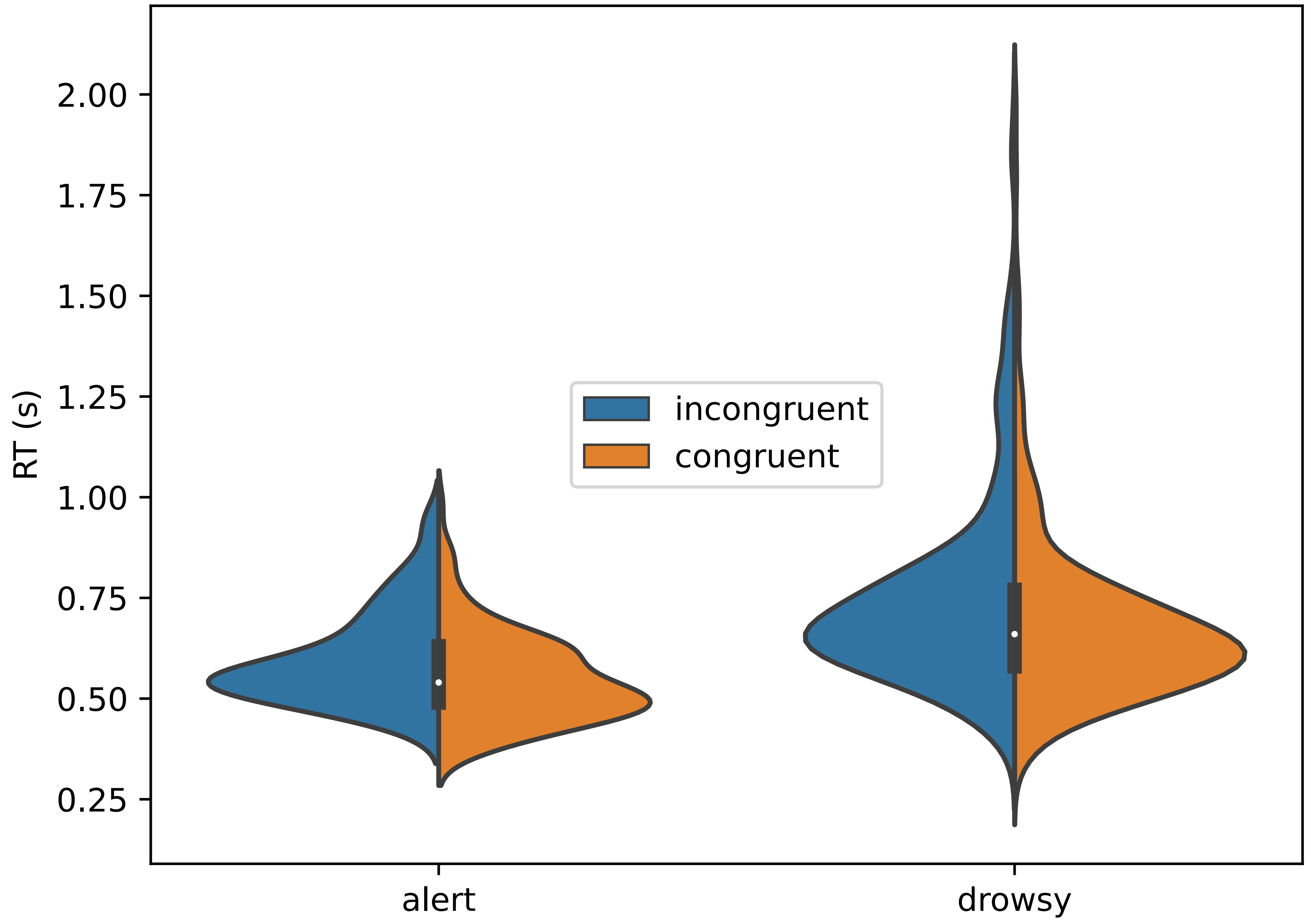
Alertness

- Good evidence this effect occurs in more than 2/3rds of the population
- Also evidence that this doesn't occur in everyone
- Everyone who has a significant effect is slower in the drowsy condition
- The overall average RT slowdown is 207ms (LMEM), but ~84% of the population would show a significant RT slowdown in this experiment



Information Theory

- Non-parametric, robust, within-participant statistical tests
- Bin RT within-participant: top 3rd, middle 3rd, bottom 3rd of RT



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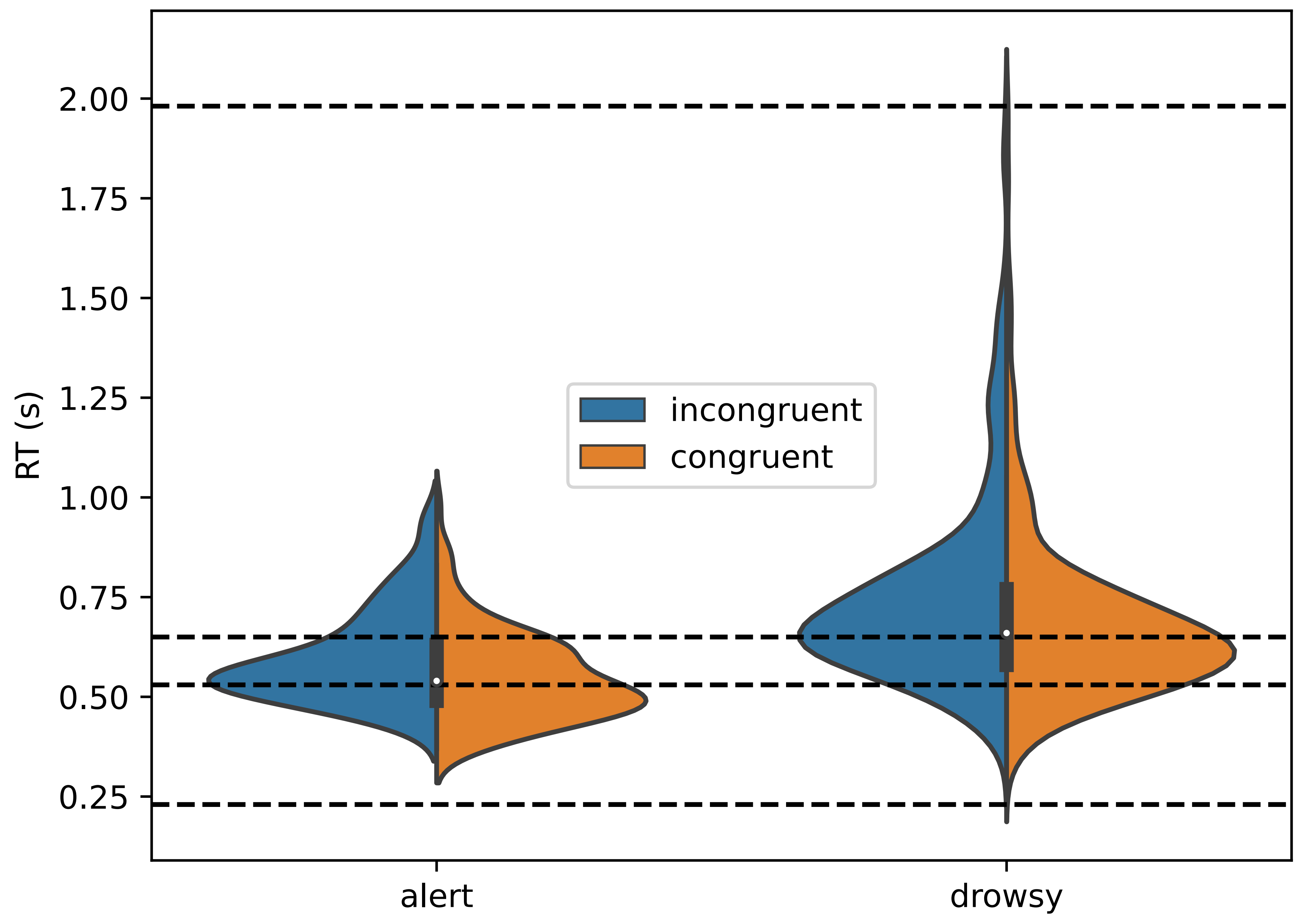

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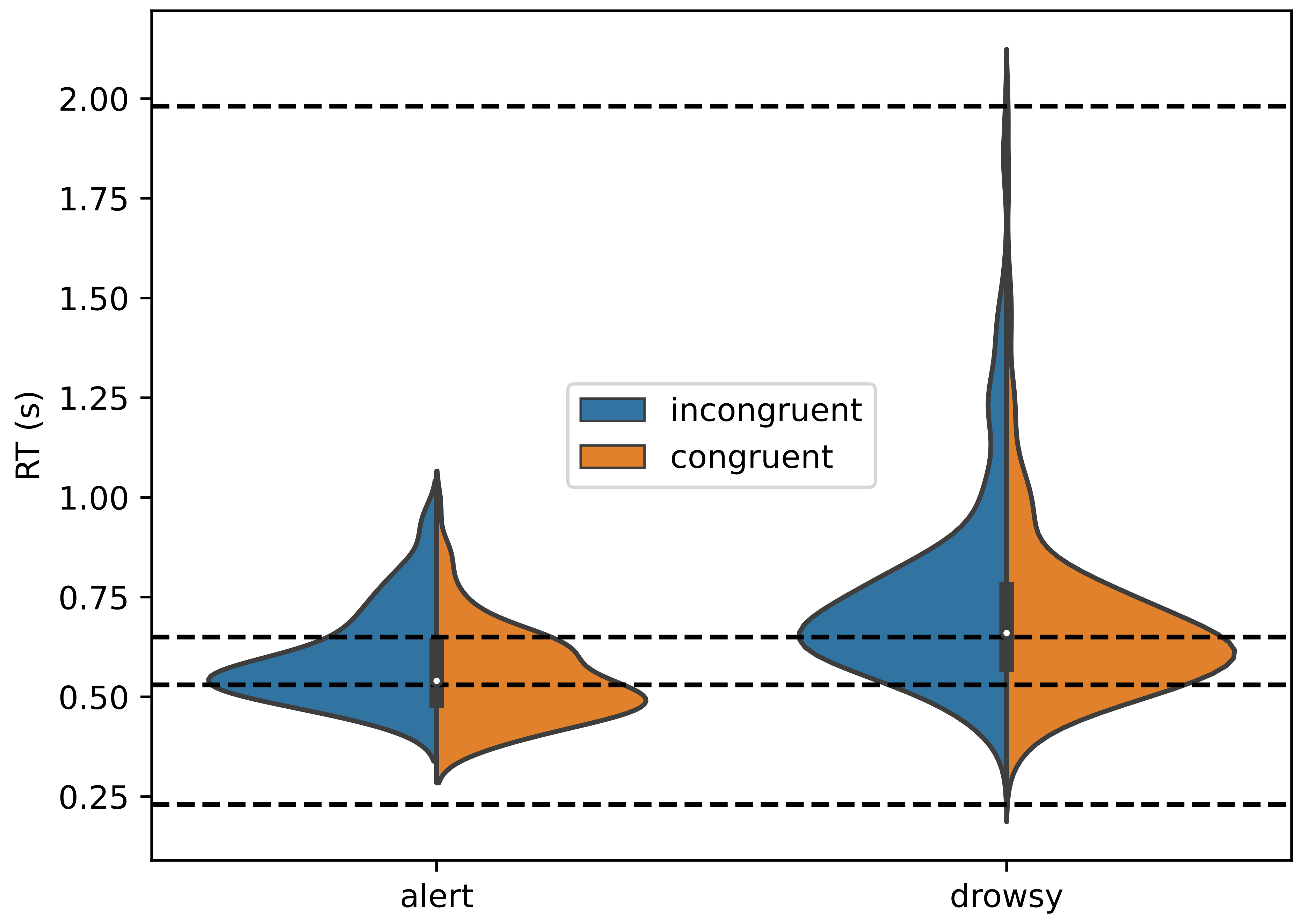

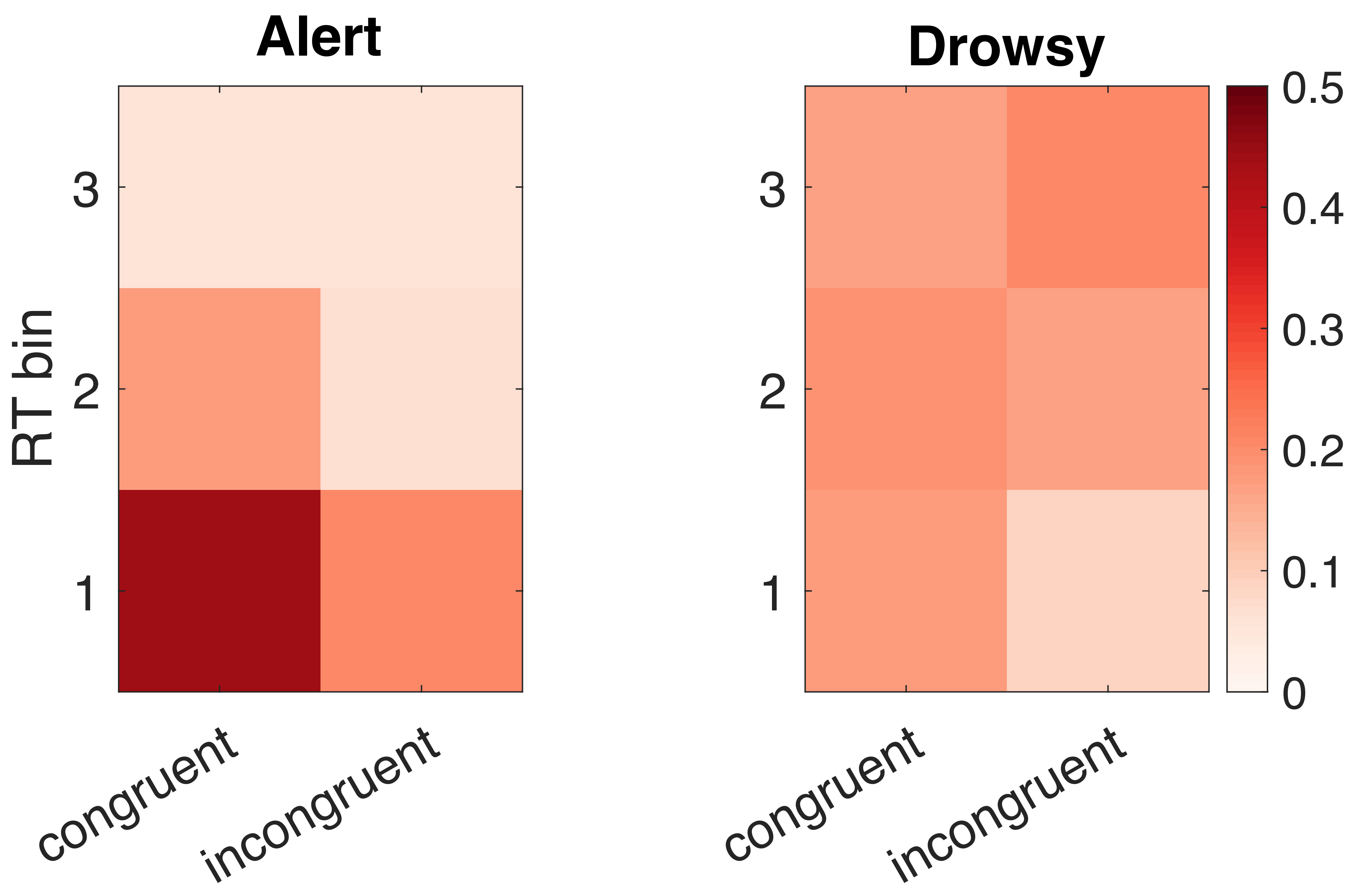

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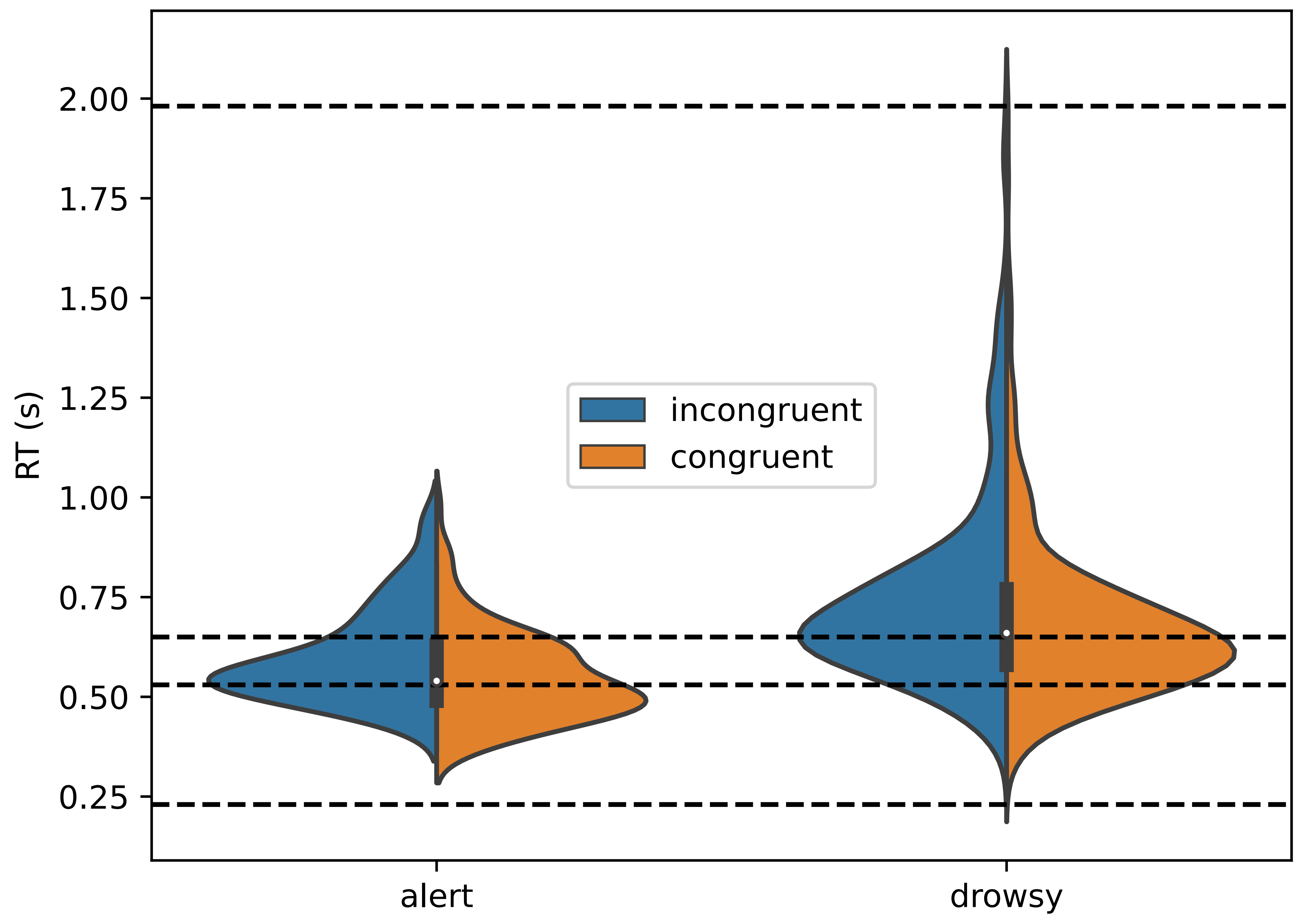

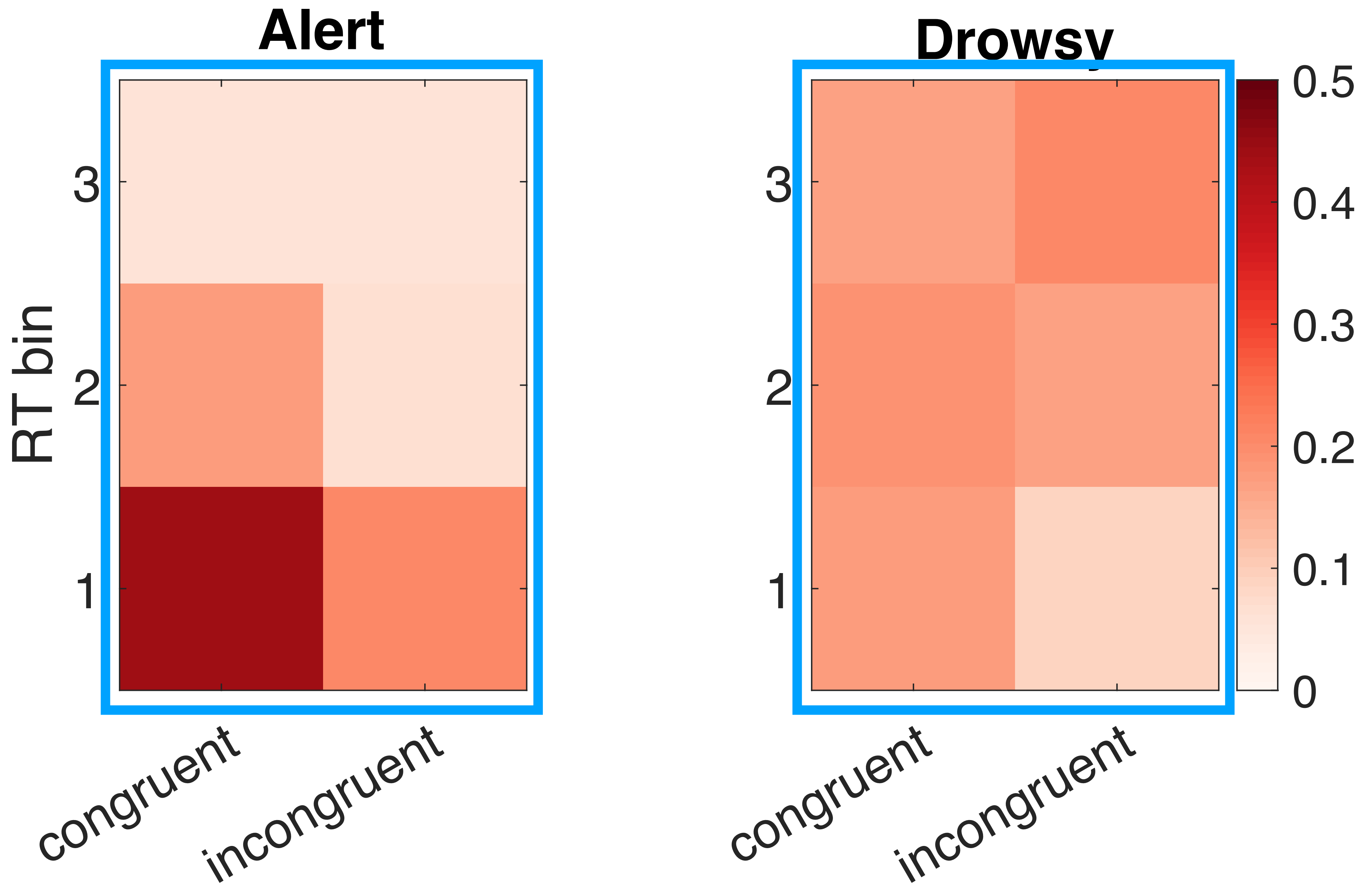

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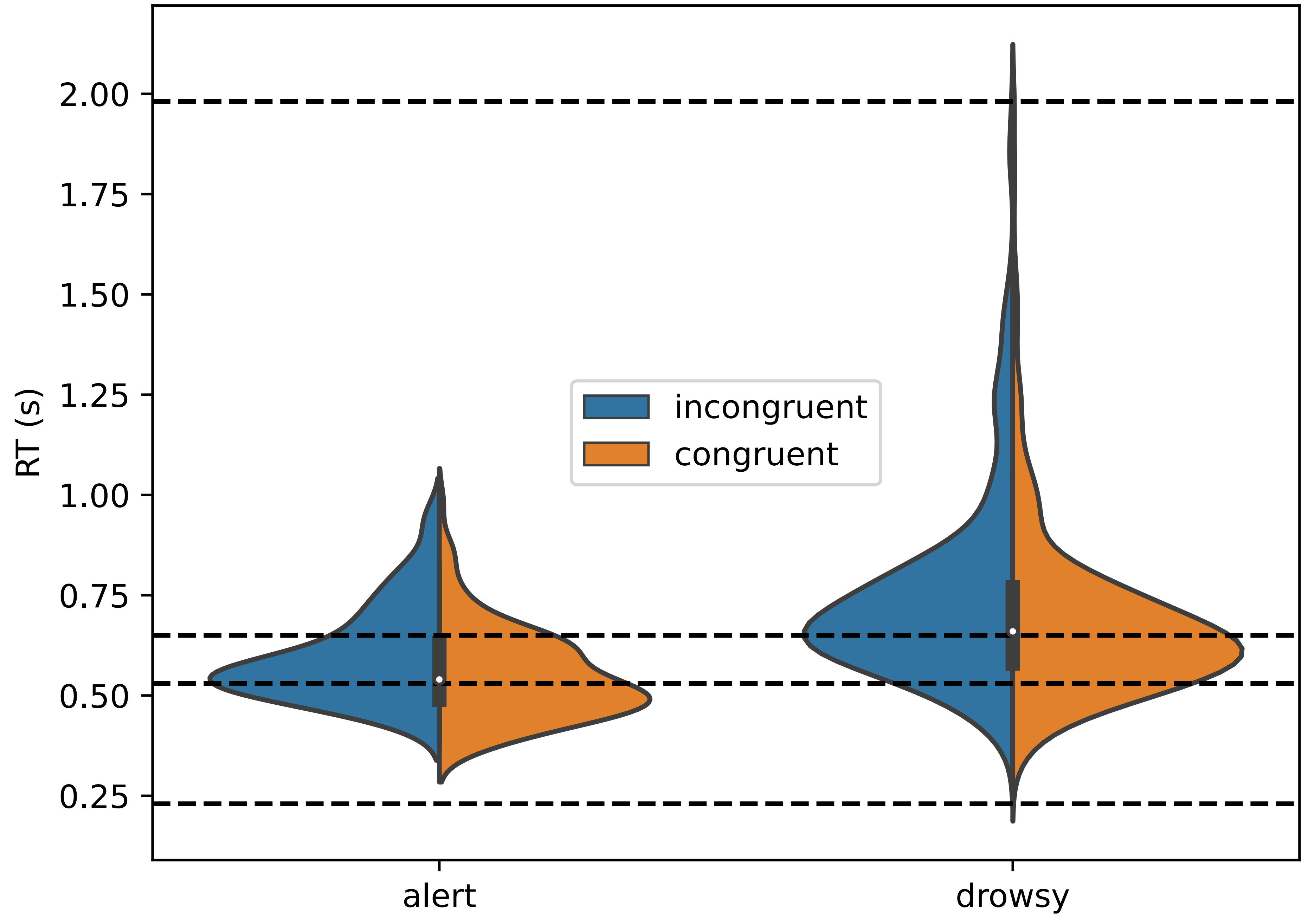
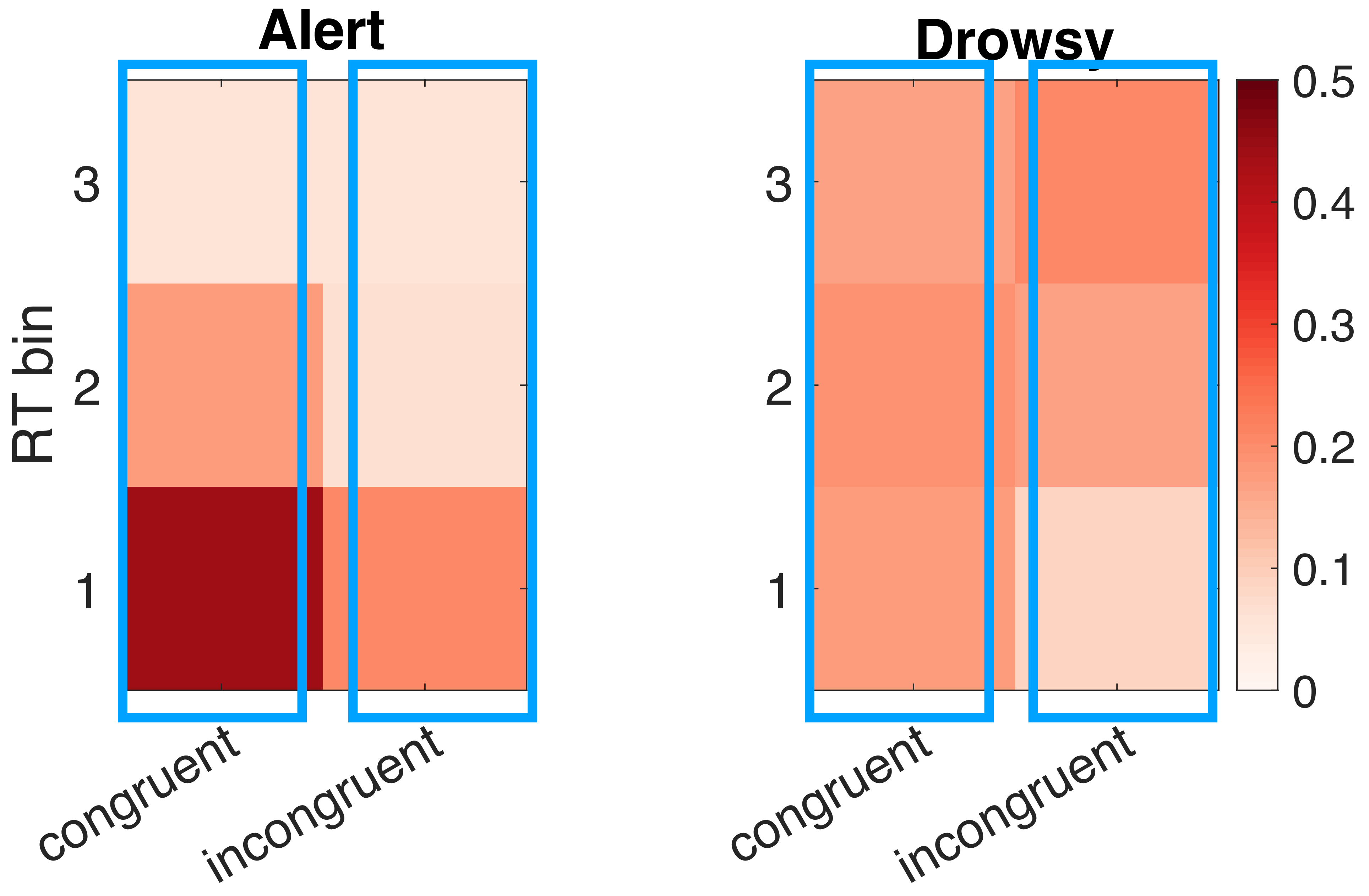
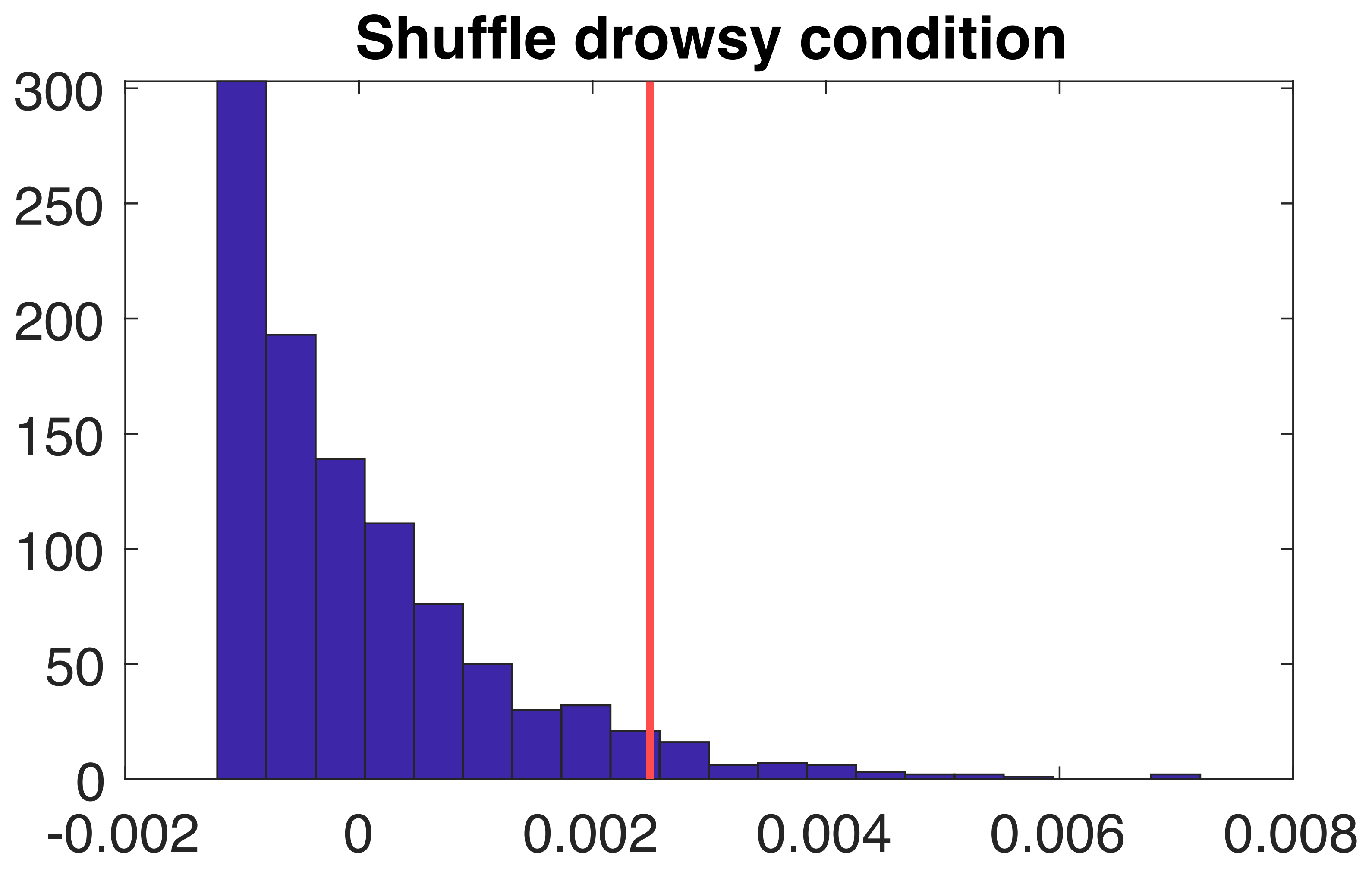




Non-parametric permutation test

- Only assumes independence of trials
- Shuffle **drowsy** condition label



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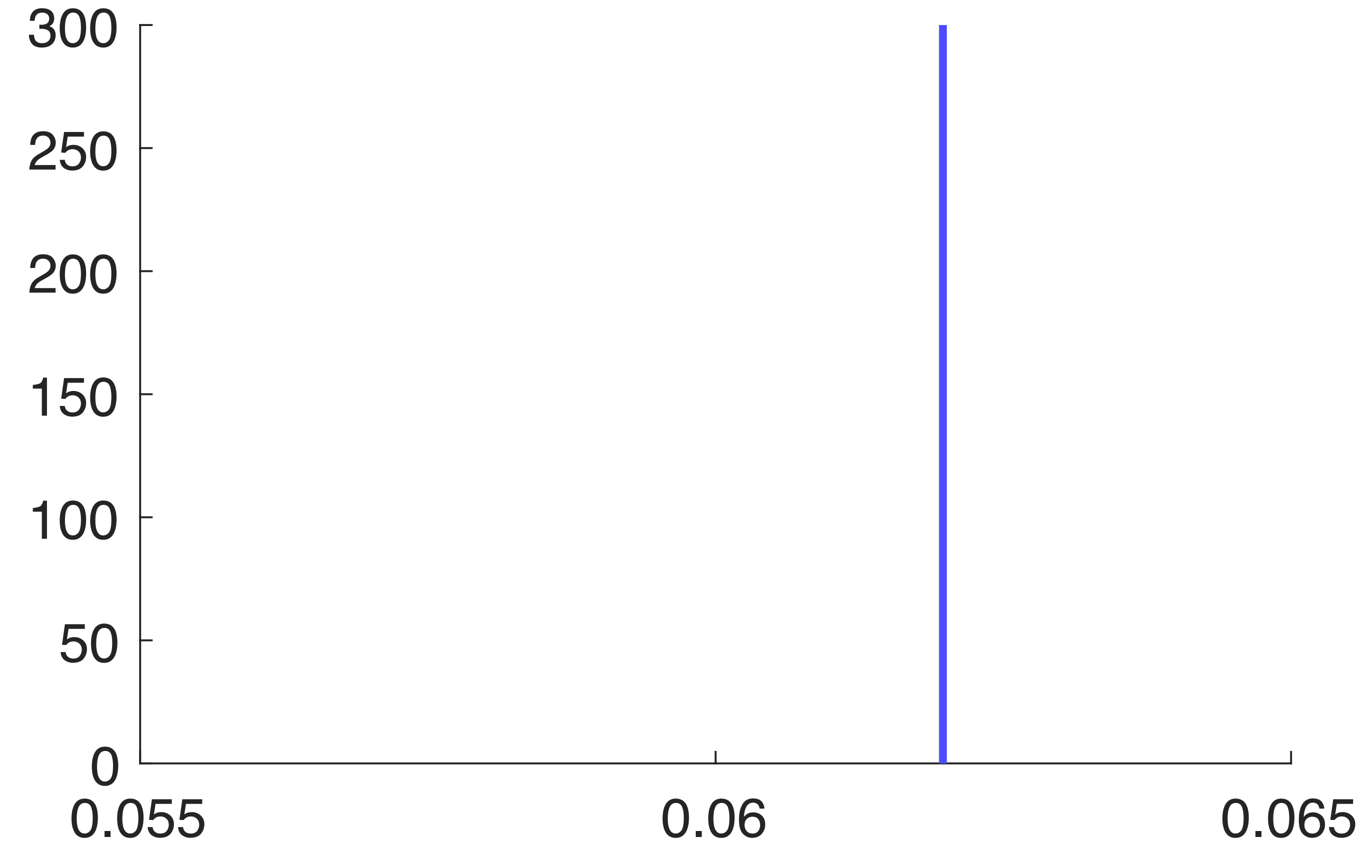
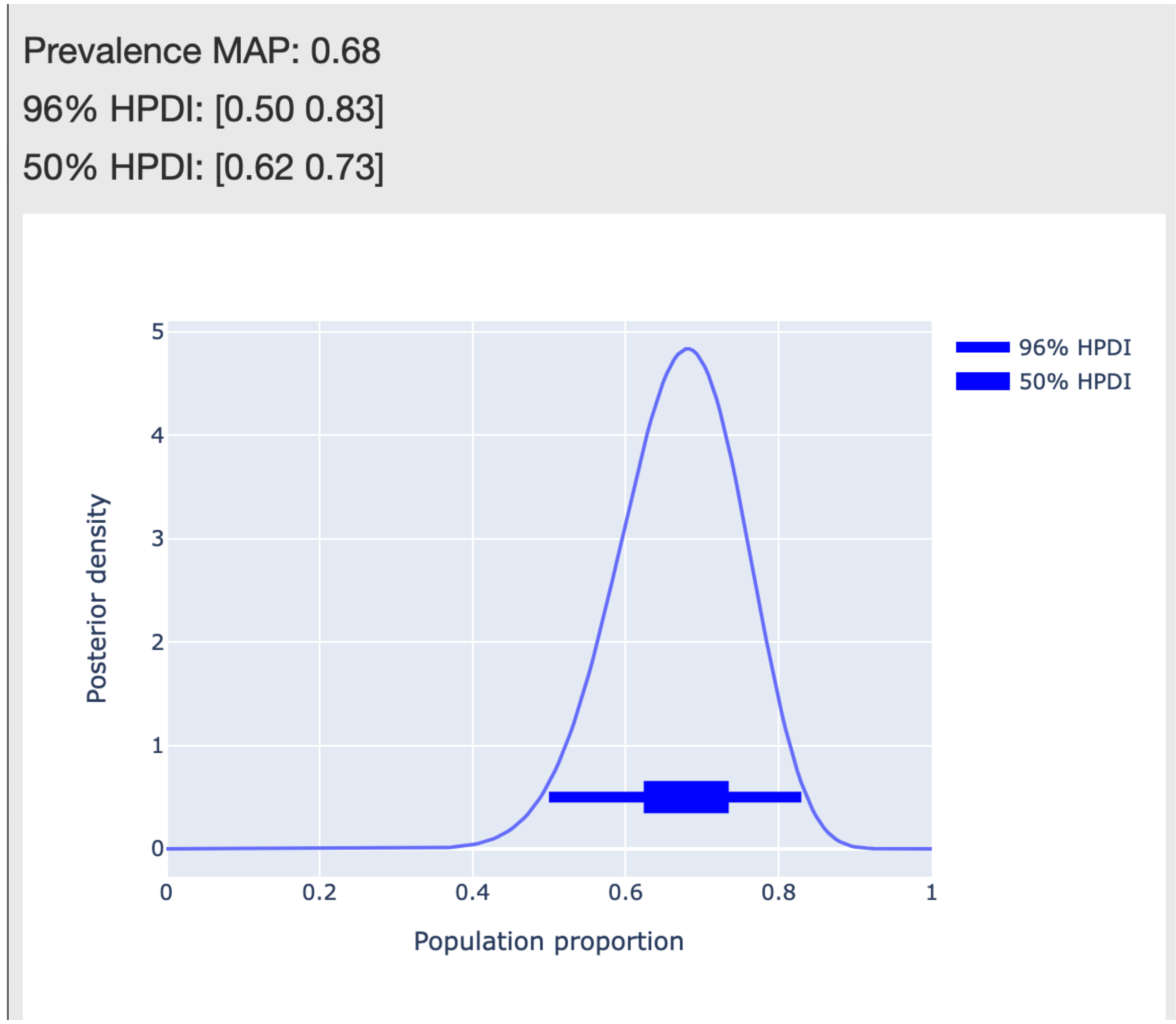


Within-participant MI

- **Congruence** : MI(congruence; RT) : 23/33
  Can detect congruence effect in majority of population with this experiment
  (LM: 5/33 or 11/33 with logRT)



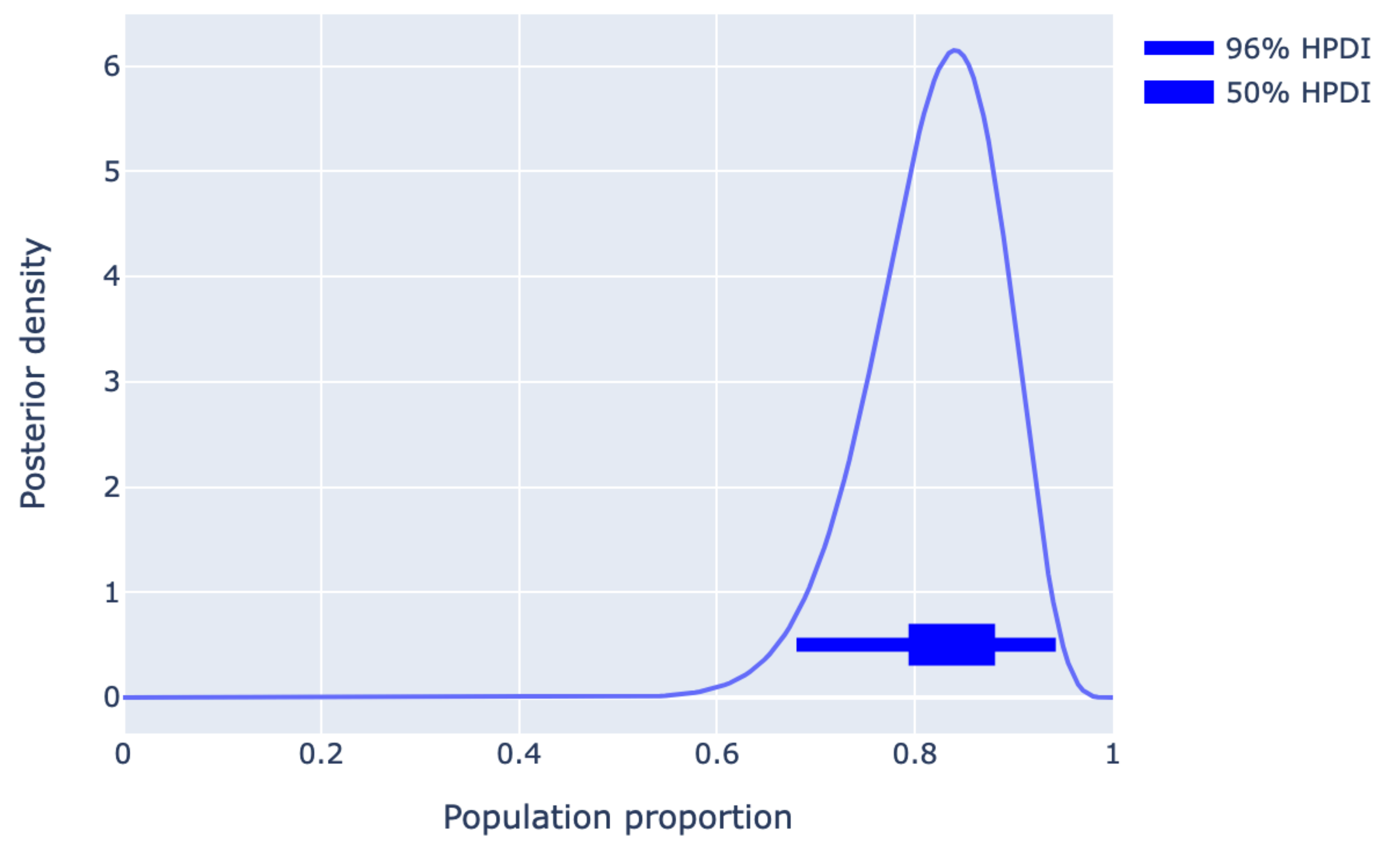
Within-participant MI

- **Congruence** : MI(congruence; RT) : 23/33
  Can detect congruence effect in majority of population with this experiment
  (LM: 5/33 or 11/33 with logRT)

- **Alertness**: MI(alertness; RT) : 28/33
  (LM: 28/33 or 29/33 with logRT)

Prevalence MAP: 0.68
96% HPDI: [0.50 0.83]
50% HPDI: [0.62 0.73]

Prevalence MAP: 0.84
96% HPDI: [0.68 0.94]
50% HPDI: [0.79 0.88]



Within-participant Interaction

- co-Information: coI(RT; congruence; alertness)
  = MI(RT; congruence) - CMI(RT; congruence | alertness)
- **modulation index** for effect of **alertness** on the *relationship* between **RT and congruence**
- permutations: shuffle alertness (null distribution that alertness has no effect)

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- permutations: shuffle alertness (null distribution that alertness has no effect)

- 22/33 have a significant interaction!

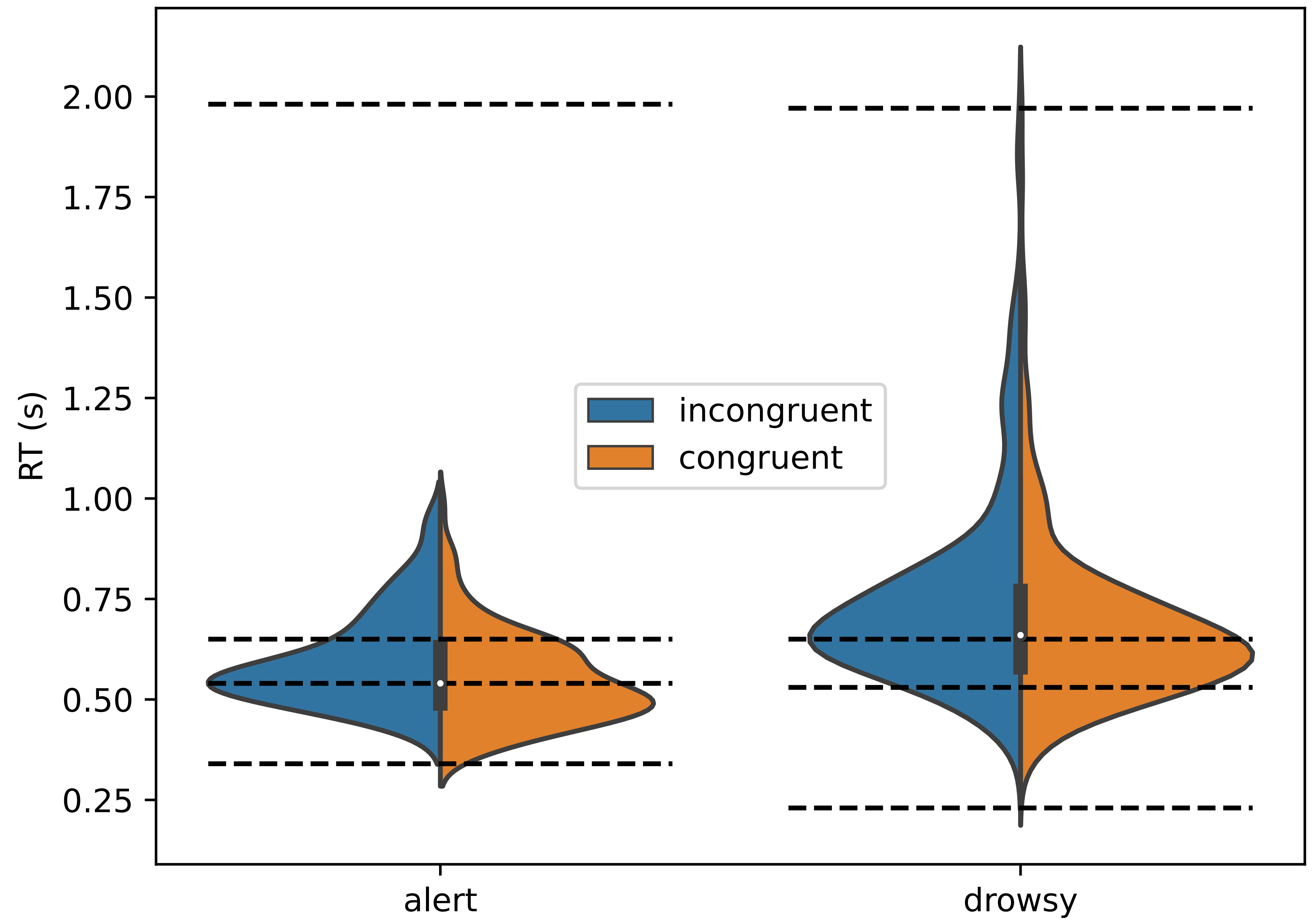
Prevalence MAP: 0.65
96% HPDI: [0.47 0.80]
50% HPDI: [0.59 0.70]

Within-participant interaction

Normalising within alertness condition

- There is a strong effect of drowsiness. Could this interaction be due to different rank ordering across the conditions because of the fixed effect of drowsiness?

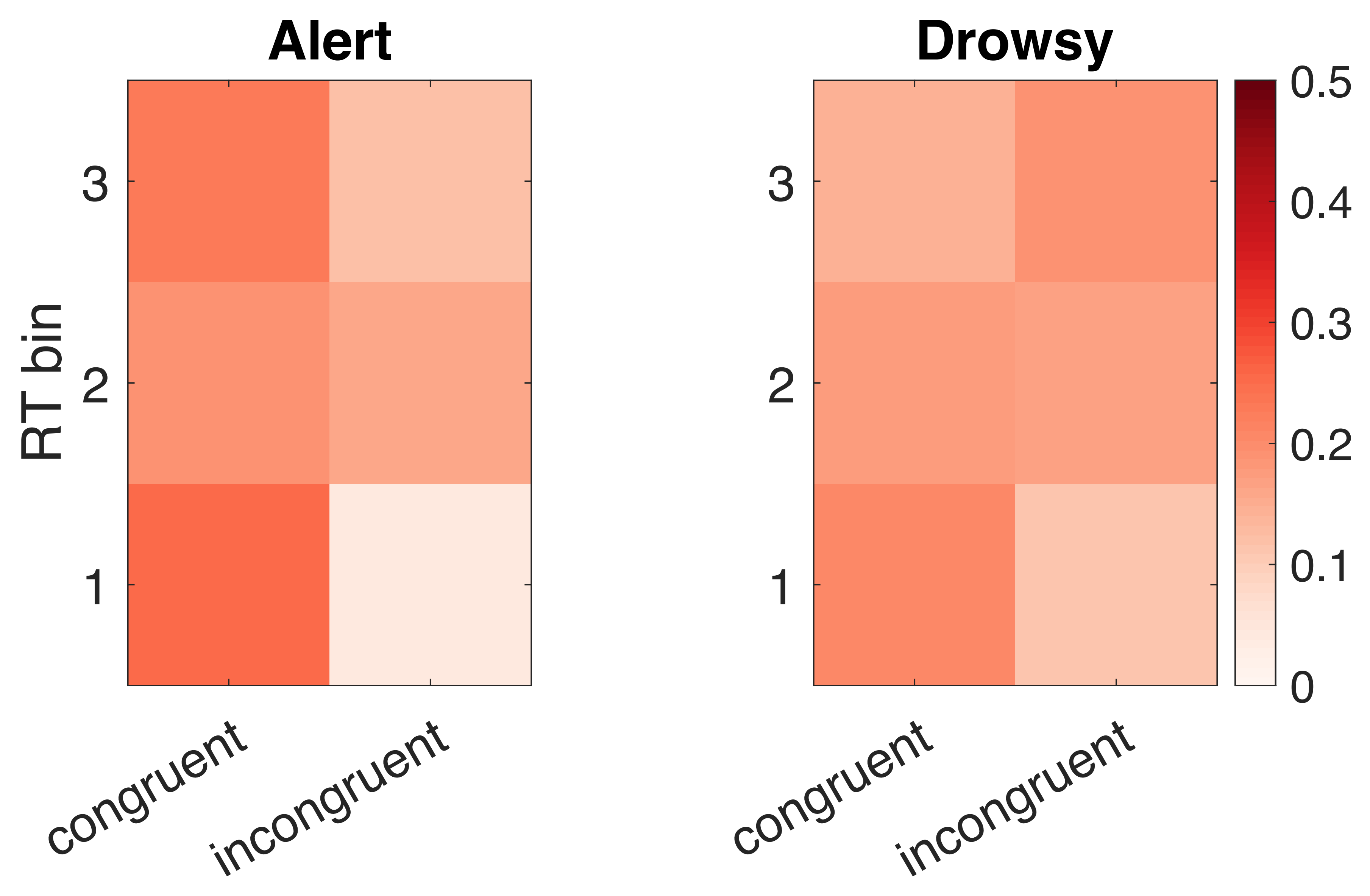
- Can bin RT in each condition separately. This is like z-scoring in each condition: both now have same uniform marginal distribution.



Within-participant interaction

Normalising within alertness condition

- No difference in marginal distribution between drowsy and alert. Effect of congruence visible in both conditions
- **Alertness**: MI(alertness; RT) : 0/33
  (we have normalised away effect of alertness)
- **Congruence** : MI(congruence; RT) : 26/33
- **Interaction**: coI(RT; congruence; alertness) : 25 /33



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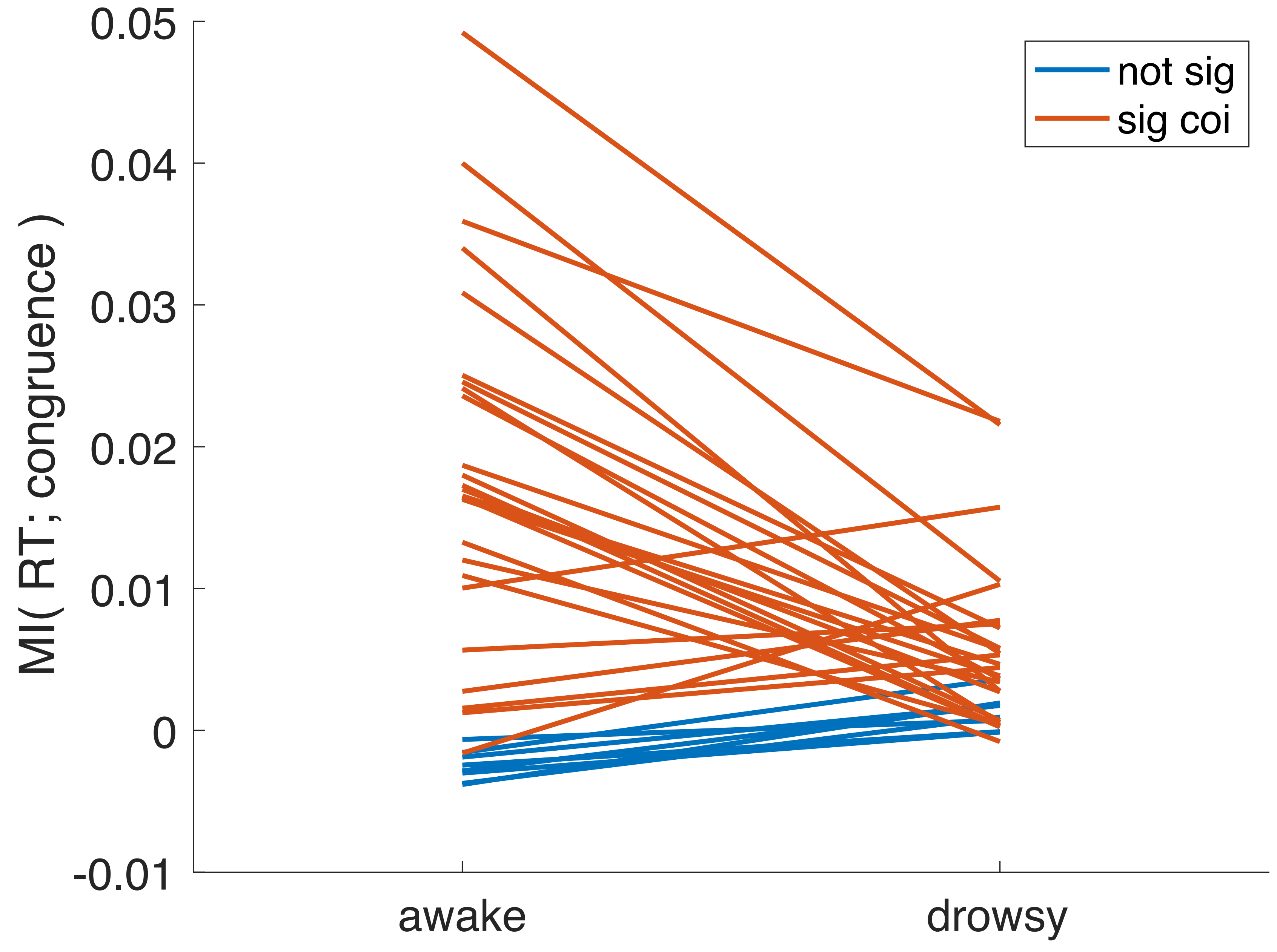
- **Interaction**: coI(RT; congruence; alertness) : 25 /33
- Interaction remains even when normalising away marginal differences between alertness conditions.
- It is a difference in the mapping between congruence and RT rank, within each condition

Within-participant interaction

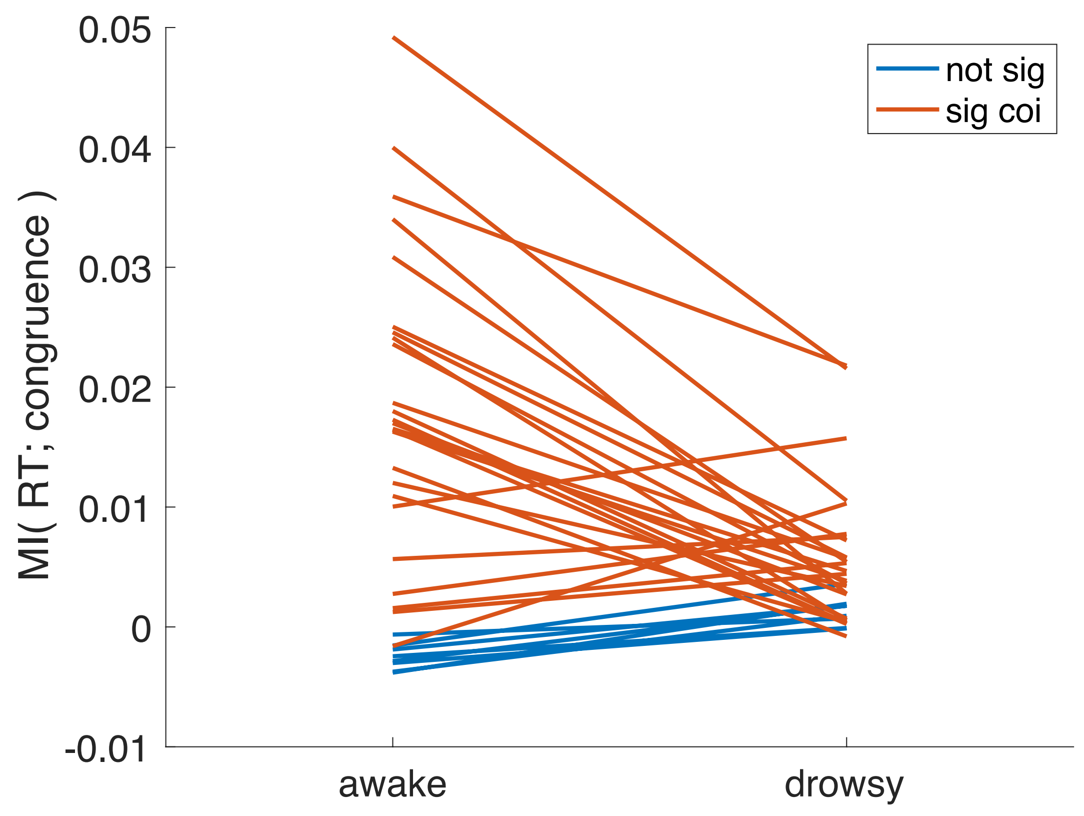
Normalising within alertness condition

- **Interaction**: coI(RT; congruence; alertness) : 25 /33
- Interaction remains even when normalising away marginal differences between alertness conditions.
- It is a difference in the mapping between congruence and RT rank, within each condition
- Majority of participants with sig. coI have weaker effect of congruence on RT in the drowsy condition



Within-participant interaction

Normalising within alertness condition

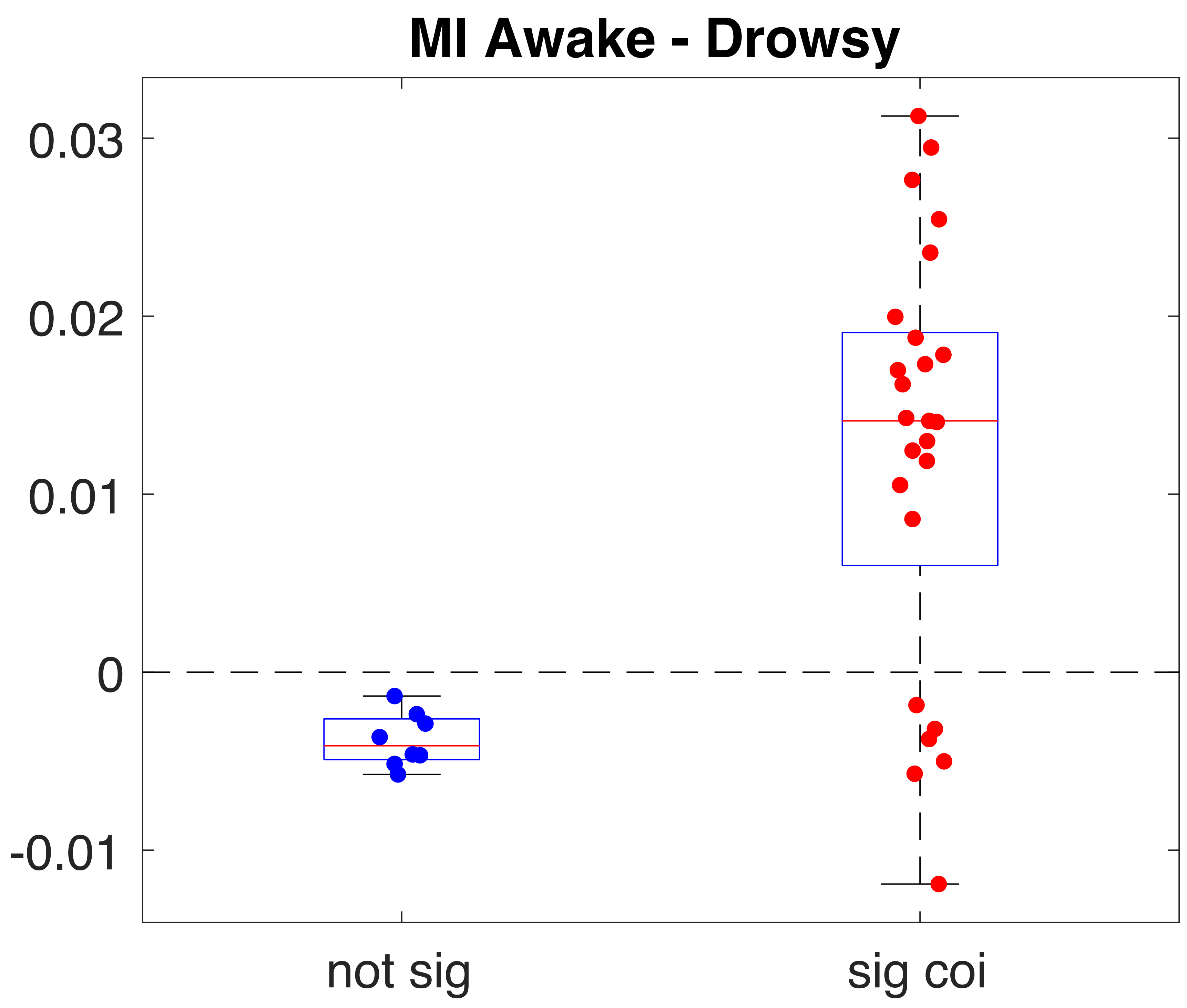


- **Interaction**: coI(RT; congruence; alertness) : 25 /33

- Majority of participants with sig. coI have lower MI(RT; congruence) in the drowsy condition

- This means: can better predict congruence condition from observing reaction times when they are awake

- I.e. 0.05 bits : need to observe 20 trials on average to predict congruence
  0.02 bits : need to observer 50 trials to predict congruence



Within-participant interaction

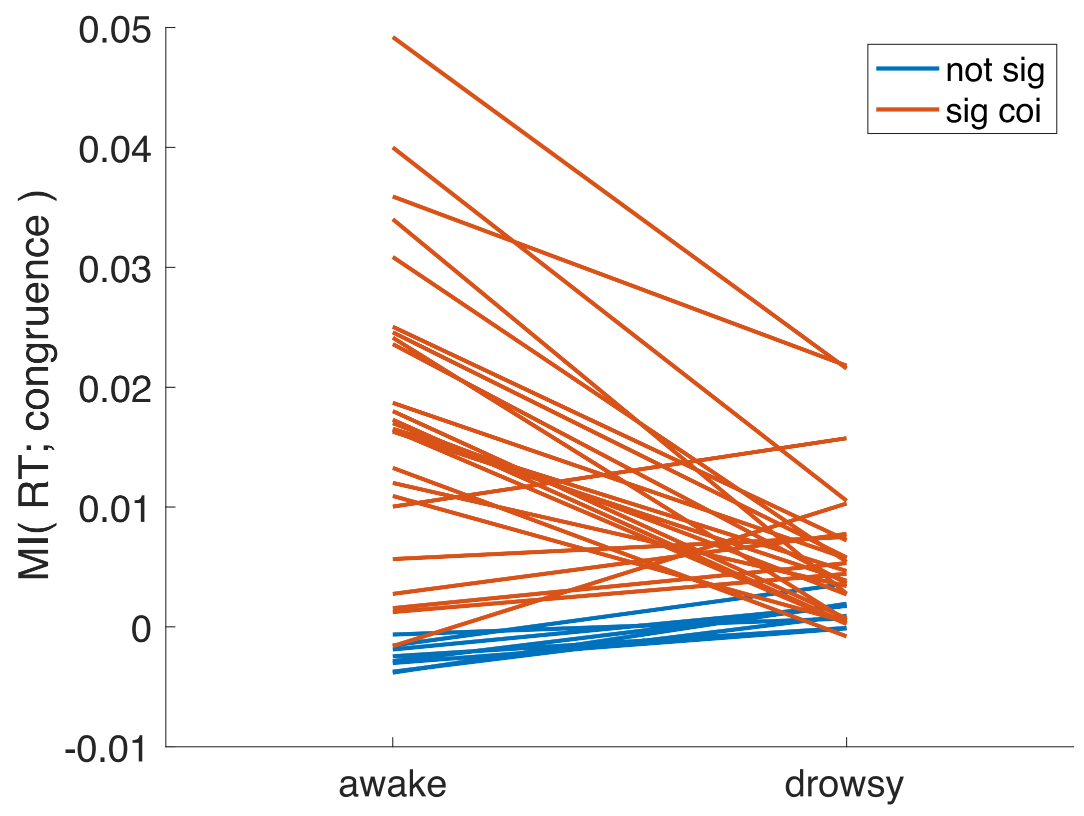
Normalising within alertness condition

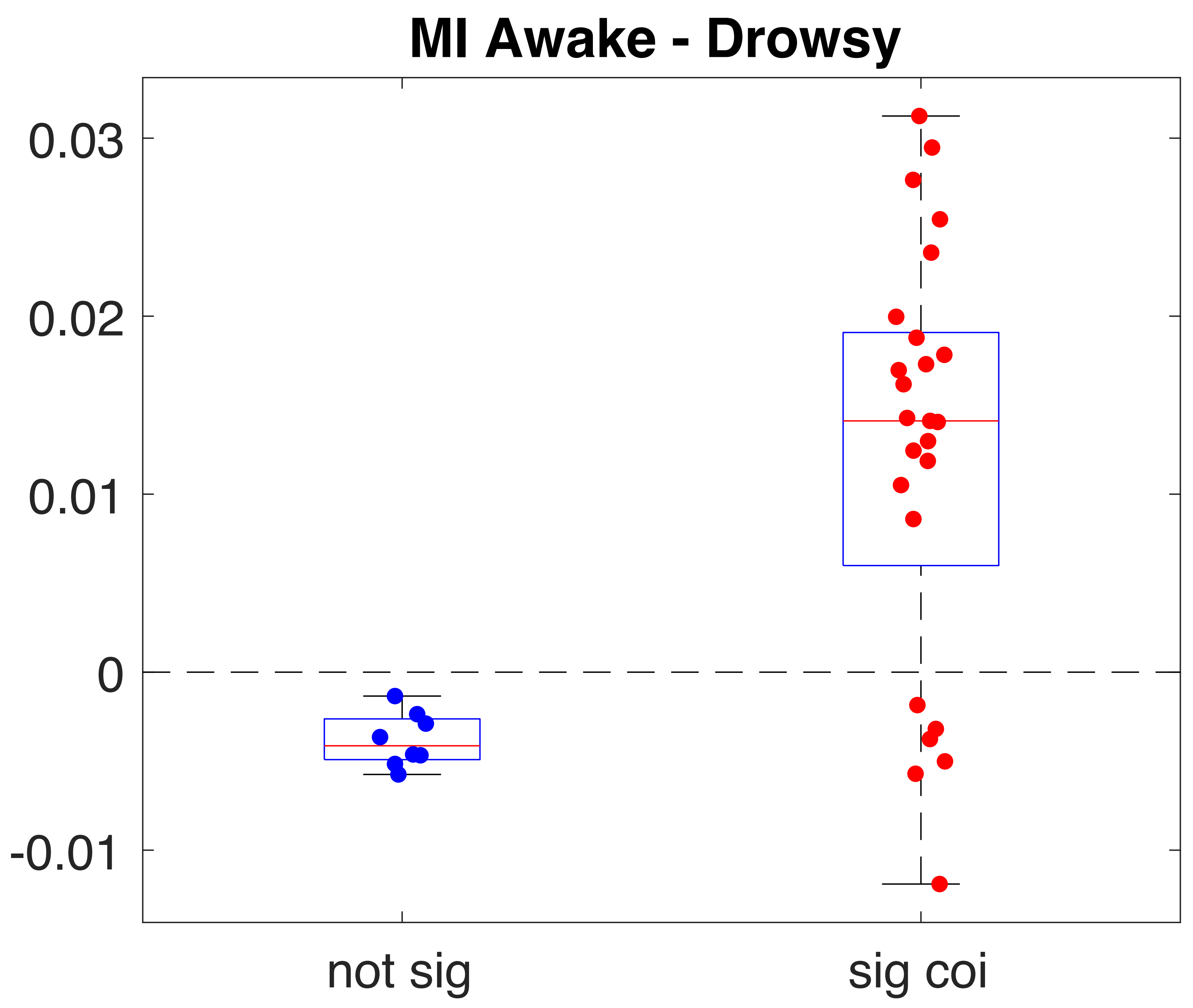
- **Interaction**: coI(RT; congruence; alertness) : 25 /33

- Majority of participants with sig. coI have lower MI(RT; congruence) in the drowsy condition

- But not all!
  One has more MI in drowsy condition, and another group have differences the same range as participants without an interaction

- So what is coI measuring here?





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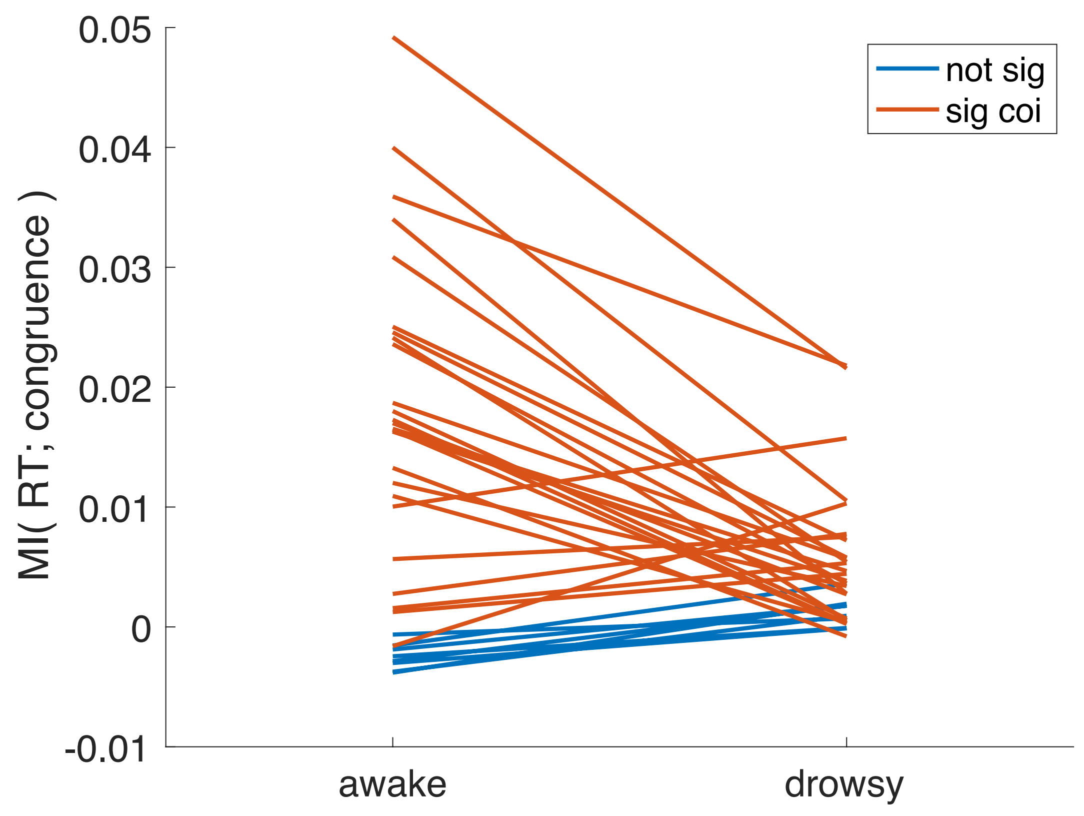
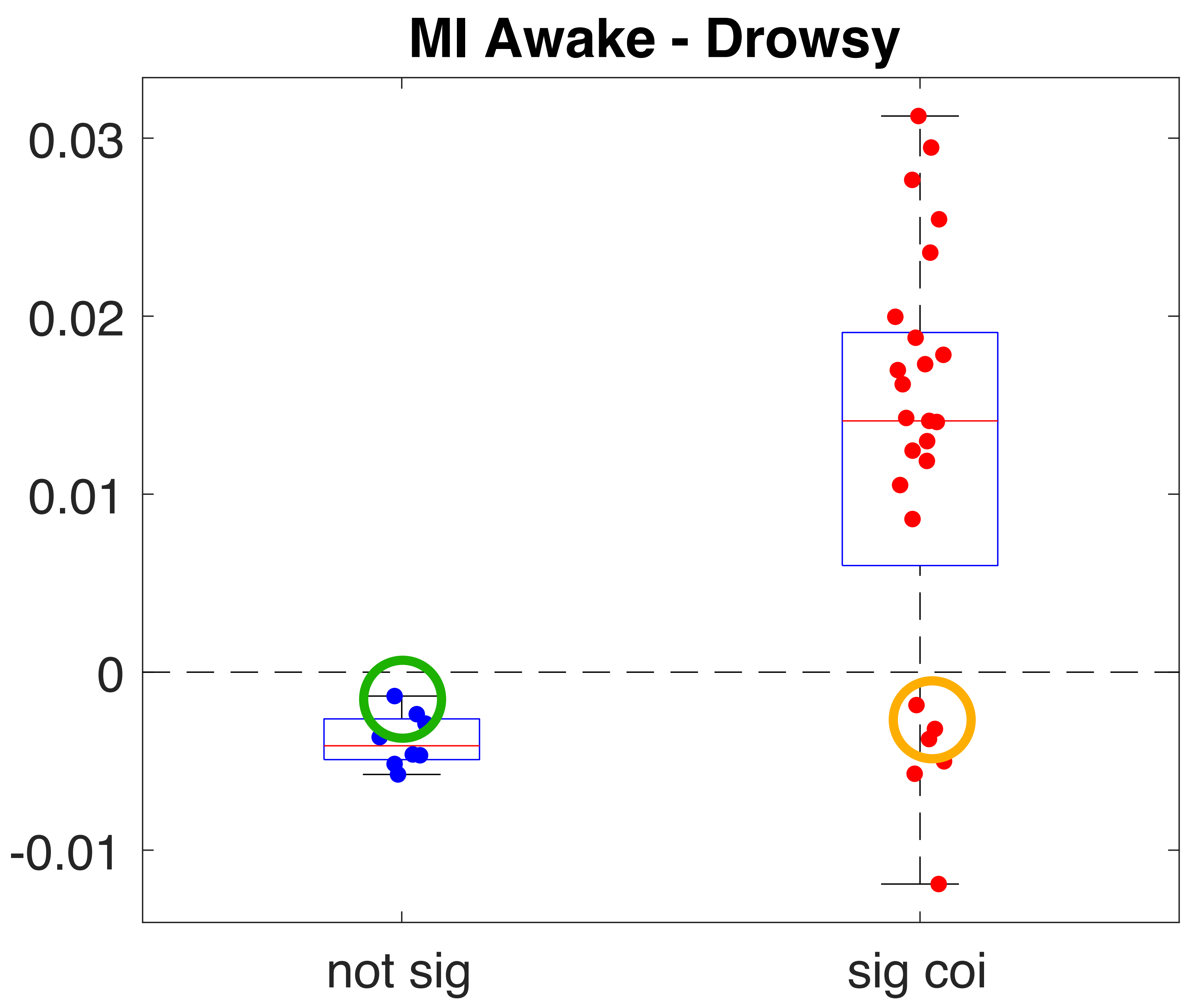


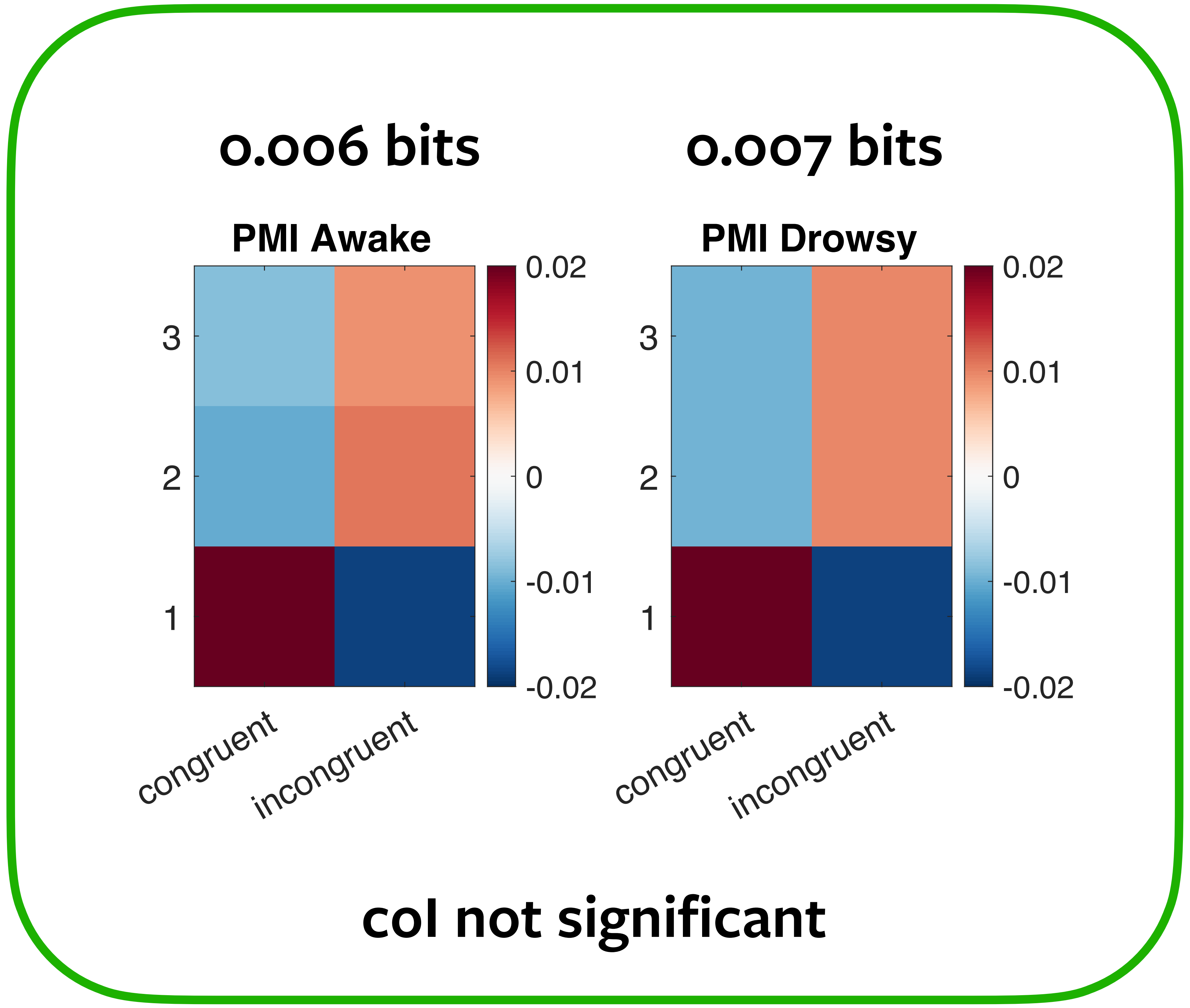


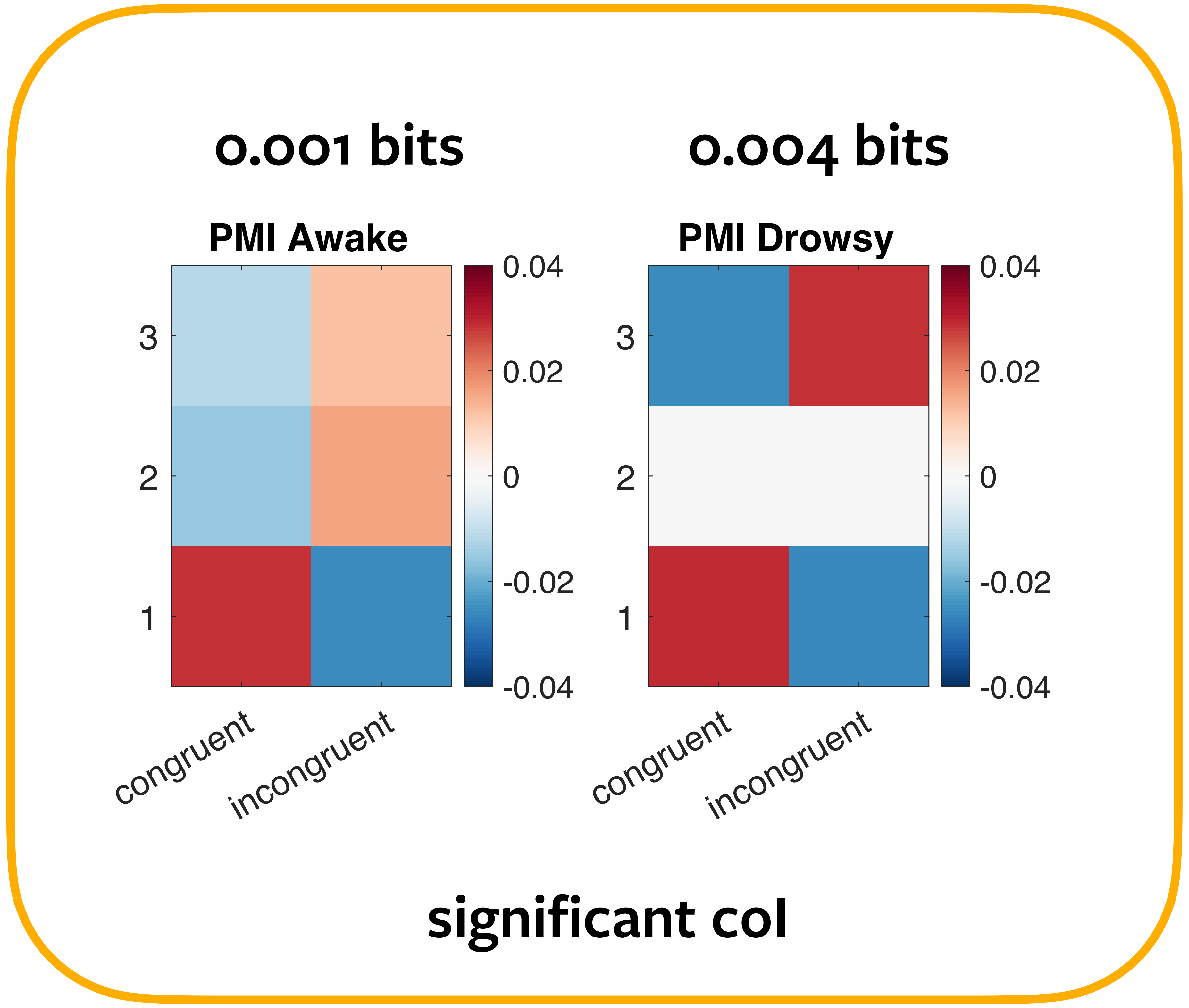
Within-participant Interaction

What is coI measuring for those participants with no MI difference?

- Breakdown MI(RT; congruence) into the contributions of each combination : pointwise mutual information (PMI)

I(RT; congruence)



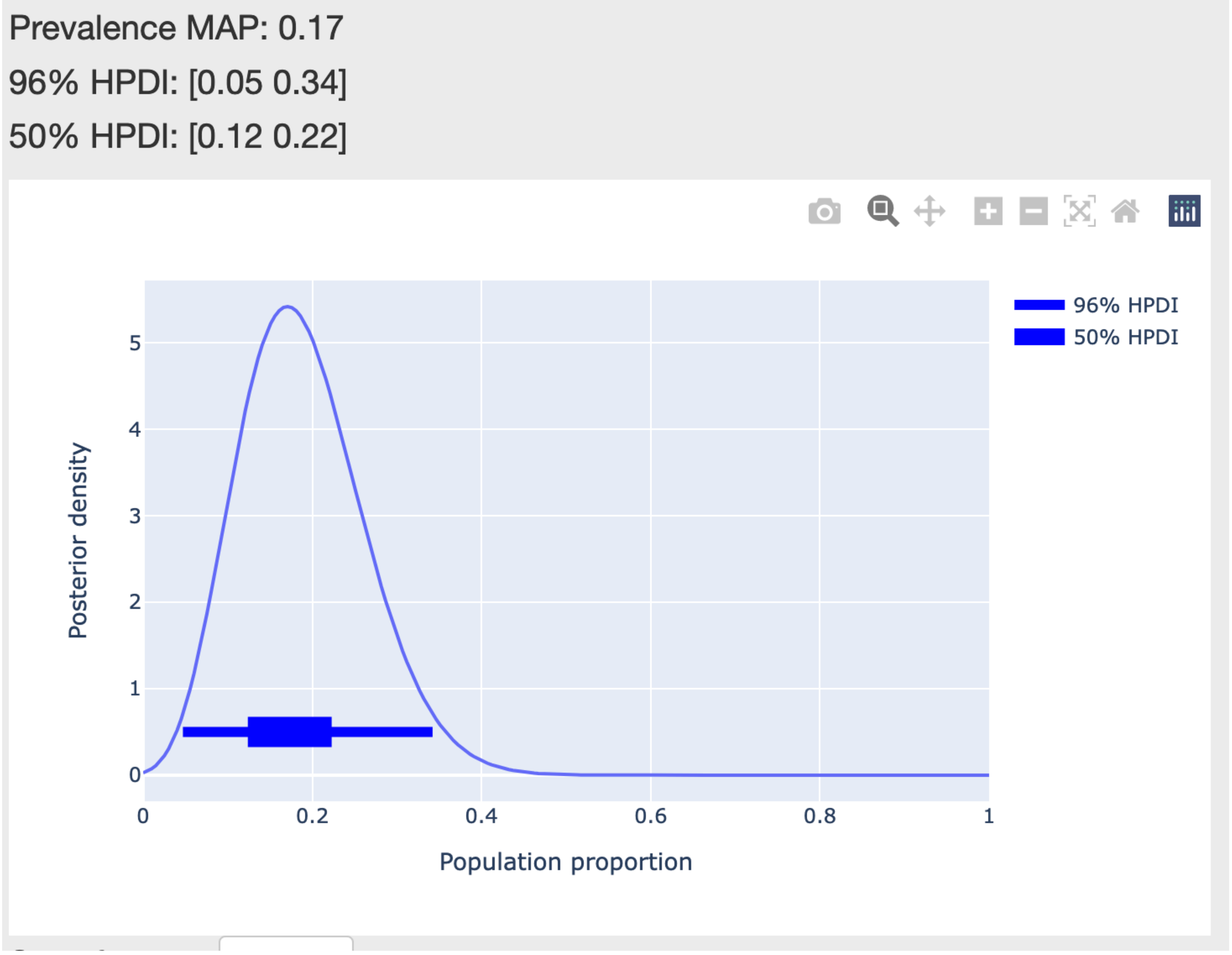


Population mean theta power

```
Linear mixed-effects model fit by ML

Model information:
    Number of observations           31494
    Fixed effects coefficients           4
    Random effects coefficients        132
    Covariance parameters               11

Formula:
    RT ~ 1 + congruent*drowsy + (1 + congruent*drowsy | ID)

Model fit statistics:
    AIC                BIC                LogLikelihood       Deviance
    186002.103418761    186127.466703719    -92986.0517093807    185972.103418761

Fixed effects coefficients (95% CIs):
    Name                          Estimate              SE                    tStat                 DF       pValue
    {'(Intercept)'     }          0.1731843078659    0.188766712004545     0.91745152536073    31490      0.358913109686981
    {'congruent'       }         0.48460173439805    0.106257660414205     4.56062868793664    31490    5.11926886598408e-06
    {'drowsy'          }       -0.711831267208987    0.120697539366188    -5.89764522911559    31490    3.72484753626096e-09
    {'congruent:drowsy'}       -0.360403111762771    0.125389902393751    -2.87425944898679    31490     0.0040524718335375
```

Within-participant theta power

- Linear Model: congruence: 6/33, alertness: 13/33, interaction: 5/33
- MI: congruence: 6/33, alertness: 18/33, co-information: 7/33
- MI approach has slightly more sensitivity within-participant but shows similar effects.
- 7/33 provides evidence of effect at population level
- Supports group level interaction, but suggests either experiment underpowered for within participant, or not everyone shows it



Within-participant theta power

Prevalence MAP: 0.17
96% HPDI: [0.05 0.34]
50% HPDI: [0.12 0.22]

Fixed effects coefficients (95% CIs):

Name	Estimate	SE	tStat	DF	pValue
{'(Intercept)' }	0.1731843078659	0.188766712004545	0.91745152536073	31490	0.358913109686981
{'congruent' }	0.48460173439805	0.106257660414205	4.56062868793664	31490	5.11926886598408e-06
{'drowsy' }	-0.711831267208987	0.120697539366188	-5.89764522911559	31490	3.72484753626096e-09
{'congruent:drowsy'}	-0.360403111762771	0.125389902393751	-2.87425944898679	31490	0.0040524718335375

Random effects covariance parameters (95% CIs):
Group: ID (33 Levels)

Name1	Name2	Type	Estimate
{'congruent:drowsy'}	{'congruent:drowsy'}	{'std' }	0.391678112624746

What does coI tell us?

- There is a statistical interaction between the alertness condition and the congruence effect
- This is more general / less specific than an interaction in a linear model for means
- Can do non-parametric statistical testing within-participant with higher power than linear modelling
- Prevalence lets us make quantitative statements about the population, based on our experiment, which may be closer to the scientific aims
- More robust statistical results (replication built in across participants)

Advantages of information theory

- Computationally efficient within-participant non-parametric tests with good statistical power
- coI can reflect different types of interaction
- Can tell you where something interesting is happening, but not exactly what it is (i.e. no mechanistic insight here!)
- But can help with multiple comparison (selecting regions for modelling), exploratory data analysis, prevalence etc.

If you want to estimate prevalence online just go to:

https://estimate.prevalence.online/

LMEM on logRT

```
Linear mixed-effects model fit by ML

Model information:
    Number of observations           54264
    Fixed effects coefficients           4
    Random effects coefficients        132
    Covariance parameters               11

Formula:
    logRT ~ 1 + congruent*drowsy + (1 + congruent*drowsy | ID)

Model fit statistics:
    AIC      BIC      LogLikelihood    Deviance
    33466    33600    -16718           33436

Fixed effects coefficients (95% CIs):
    Name                            Estimate      SE            tStat       DF       pValue        Lower
    {'(Intercept)'     }               6.4152      0.033187       193.3     54260             0       6.3501
    {'congruent'       }             0.050629     0.0056099       9.025     54260     1.854e-19     0.039634
    {'drowsy'          }              0.24012      0.034498      6.9604     54260    3.4319e-12       0.1725
    {'congruent:drowsy'}           -0.0017643     0.0067334    -0.26202     54260       0.79331    -0.014962
```