

Within-participant information theory + Prevalence Example

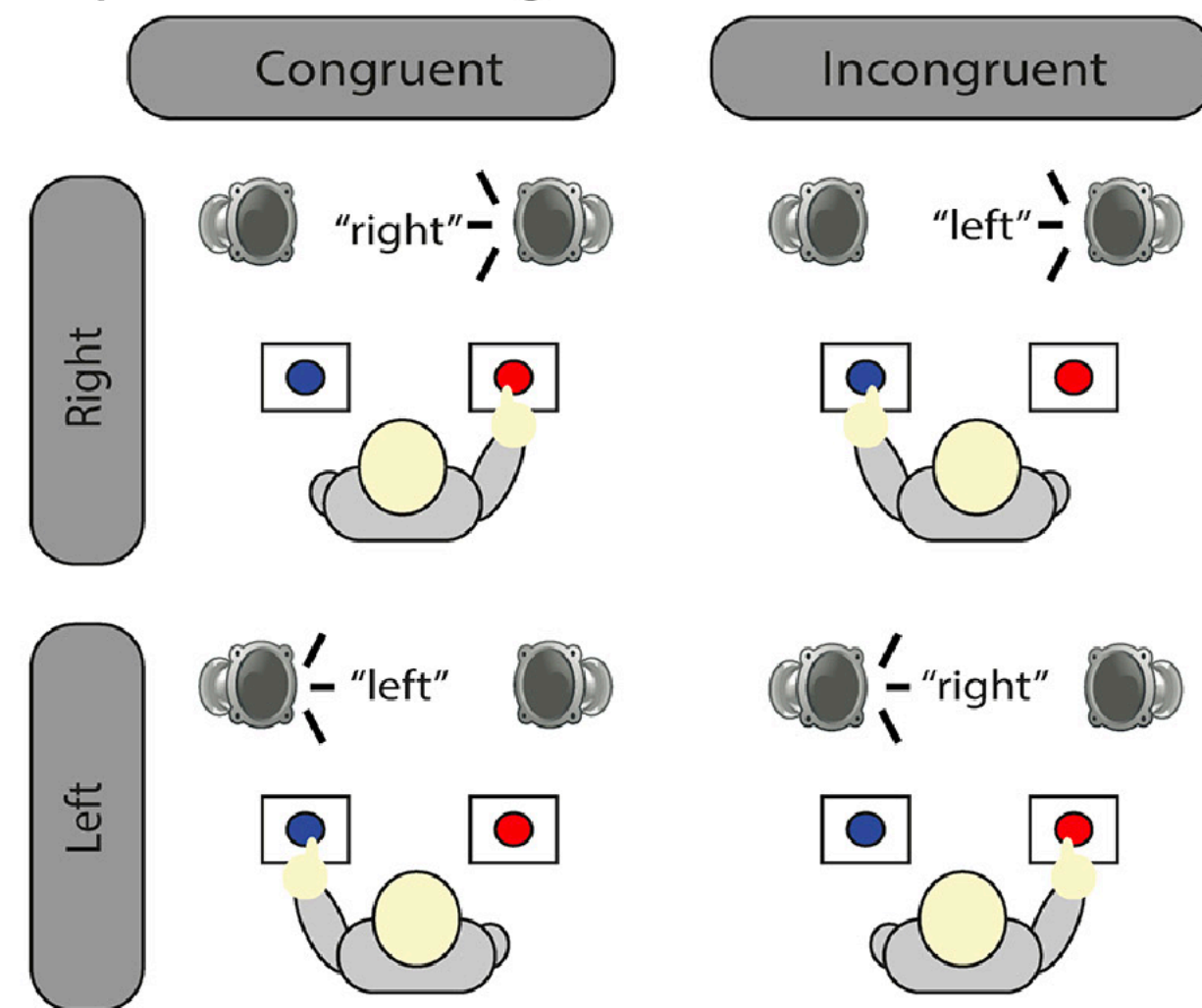
Effect of decreased alertness on cognitive control

<https://www.robince.net/data/ams2024>

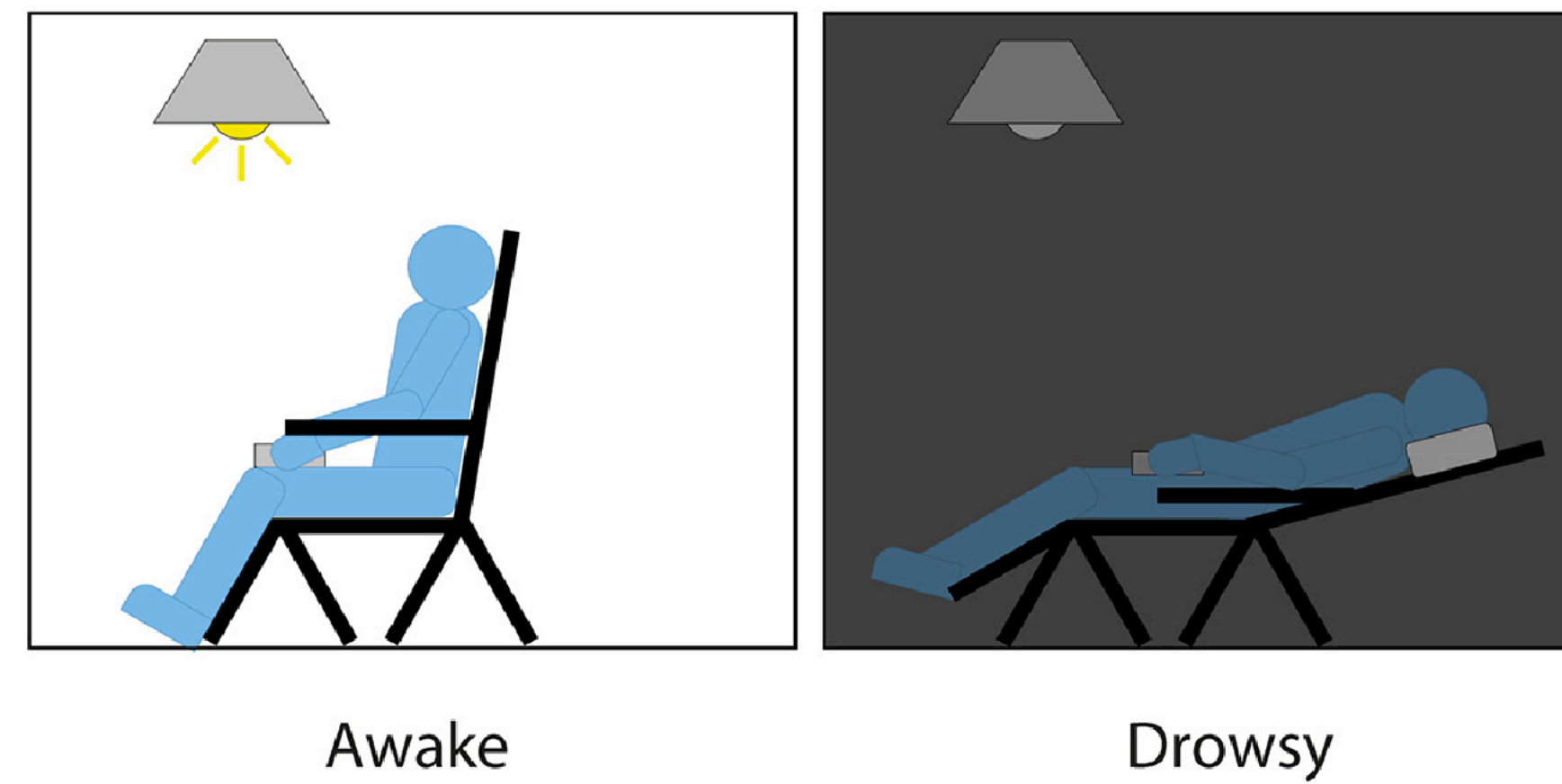
Behavioural Analysis

- 33 participants did an auditory stroop task (left vs right):
congruent and **incongruent** trials; **awake** vs **drowsy** experimental conditions
~1600 trials per participant

A Experimental design



B Experimental sessions



Population mean analysis on Reaction Times

- Linear Mixed Effect Model

```
Linear mixed-effects model fit by ML

Model information:
  Number of observations      54264
  Fixed effects coefficients      4
  Random effects coefficients  132
  Covariance parameters      11

Formula:
  RT ~ 1 + congruent*drowsy + (1 + congruent*drowsy | ID)

Model fit statistics:
  AIC      BIC      LogLikelihood      Deviance
  7.72e+05  7.7213e+05  -3.8598e+05  7.7197e+05

Fixed effects coefficients (95% CIs):
  Name      Estimate      SE      tStat      DF      pValue      Lower      Upper
  {'(Intercept)'}      645.4      22.72      28.407      54260      3.2458e-176      600.87      689.93
  {'congruent'}      30.087      4.8288      6.2308      54260      4.6732e-10      20.623      39.552
  {'drowsy'}      207.73      29.692      6.9961      54260      2.6622e-12      149.53      265.93
  {'congruent:drowsy'}      6.7345      6.169      1.0917      54260      0.27499      -5.3569      18.826
```

Population mean analysis on Reaction Times

- Linear Mixed Effect Model
- Significant main effect of congruence (30 ms)
- Significant main effect of drowsiness (207 ms)
- No interaction

Fixed effects coefficients (95% CIs):								
Name		Estimate	SE	tStat	DF	pValue	Lower	Upper
{ '(Intercept)' }		645.4	22.72	28.407	54260	3.2458e-176	600.87	689.93
{ 'congruent' }		30.087	4.8288	6.2308	54260	4.6732e-10	20.623	39.552
{ 'drowsy' }		207.73	29.692	6.9961	54260	2.6622e-12	149.53	265.93
{ 'congruent:drowsy' }		6.7345	6.169	1.0917	54260	0.27499	-5.3569	18.826

Within-participant linear model of mean RT

- We can simply run the same linear model separately within each participant:

```
%% fit within participant models
sub_mdls = cell(1,Nsub);
for subi=1:Nsub
    sub_dat = RT_dat(RT_dat.ID==subi,:);
    mdl = fitlm(sub_dat, 'RT ~ congruent*drowsy');
    sub_mdls{subi} = mdl;
end
```

Within-participant linear model of mean RT

- We can simply run the same linear model separately within each participant:
- 5/33 have a significant main effect of congruence
- 28/33 have a significance main effect of drowsiness
- 0/33 have a significant interaction

Within-participant linear model of mean RT

- What do we learn about the population from this:
- <https://estimate.prevalence.online/>

Within-participant linear model of mean RT

Congruence

- Evidence to reject the “global null” - ie we can have some confidence the effect exists
- But doesn't seem to be particularly widespread
- Perhaps experiment underpowered to detect this within participants?

Bayesian prevalence of a statistical test

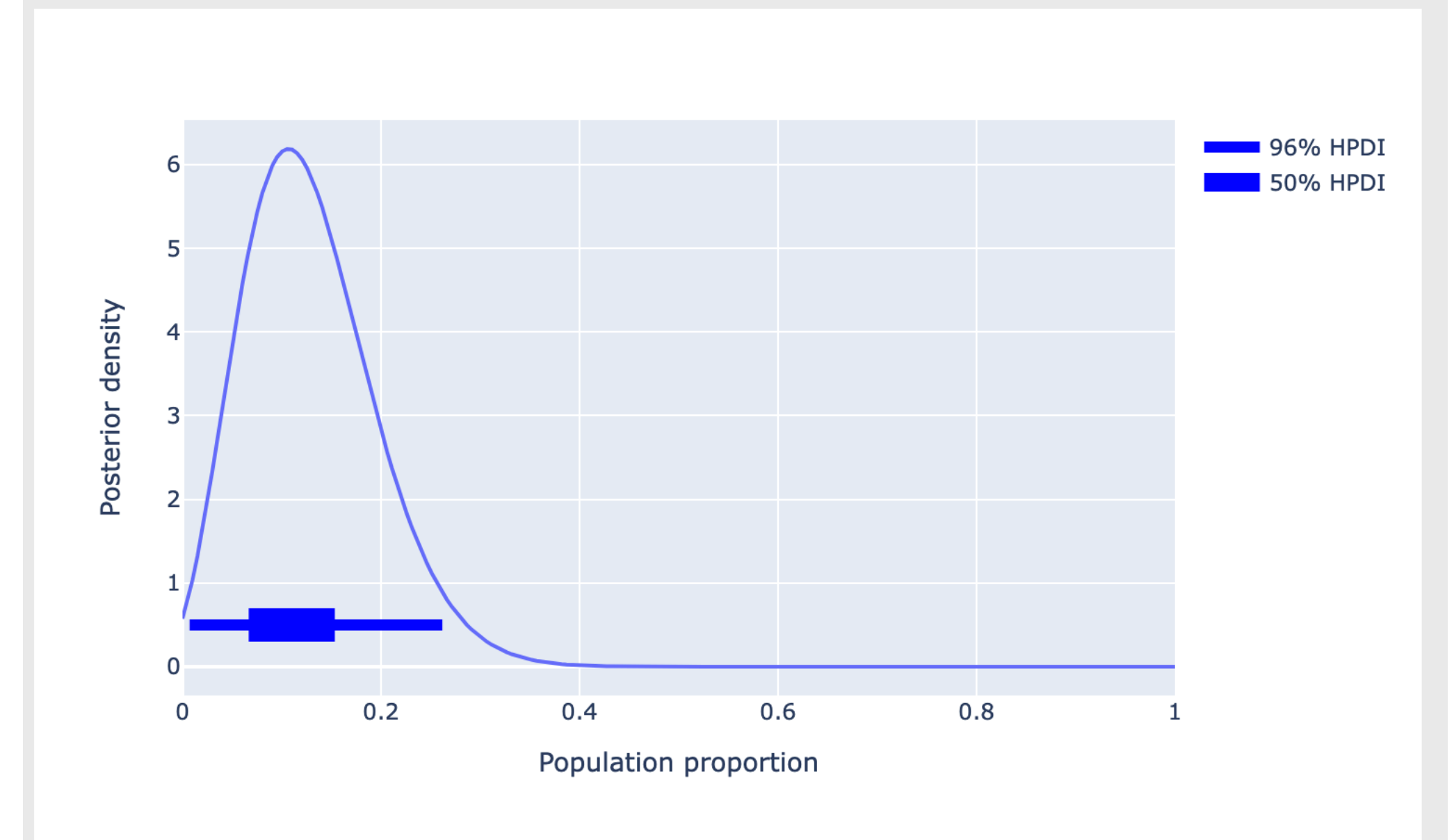
5 out of 33 tests significant with false positive rate 0.05

Bayesian results

Prevalence MAP: 0.11

96% HPDI: [0.01 0.26]

50% HPDI: [0.07 0.15]



Within-participant linear model of mean RT

Alertness

- Good evidence this effect occurs in more than 2/3rds of the population
- Also evidence that this doesn't occur in everyone
- Everyone who has a significant effect is slower in the drowsy condition
- The overall average RT slowdown is 207ms (LMEM), but ~84% of the population would show a significant RT slowdown in this experiment

Bayesian prevalence of a statistical test

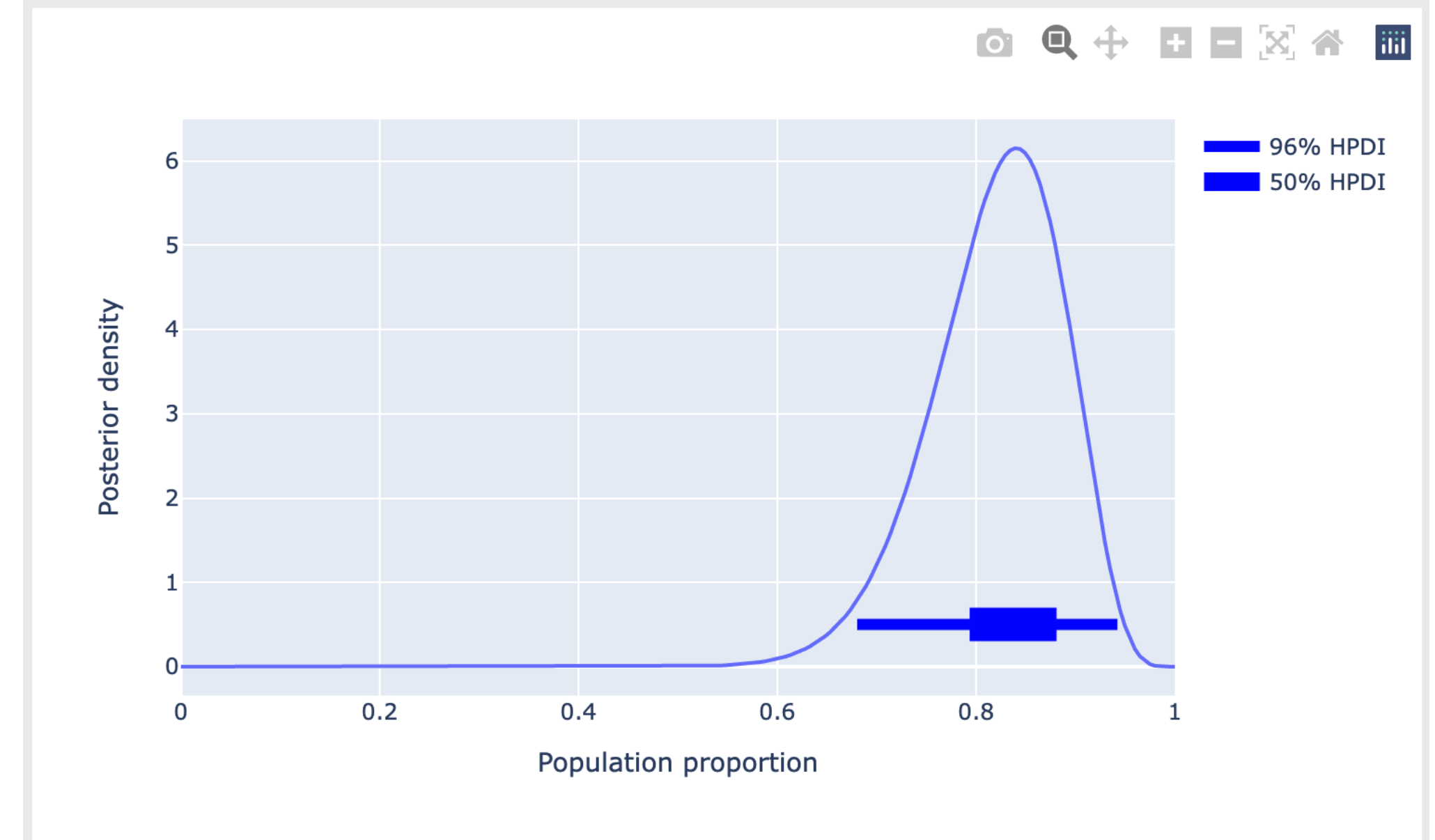
28 out of 33 tests significant with false positive rate 0.05

Bayesian results

Prevalence MAP: 0.84

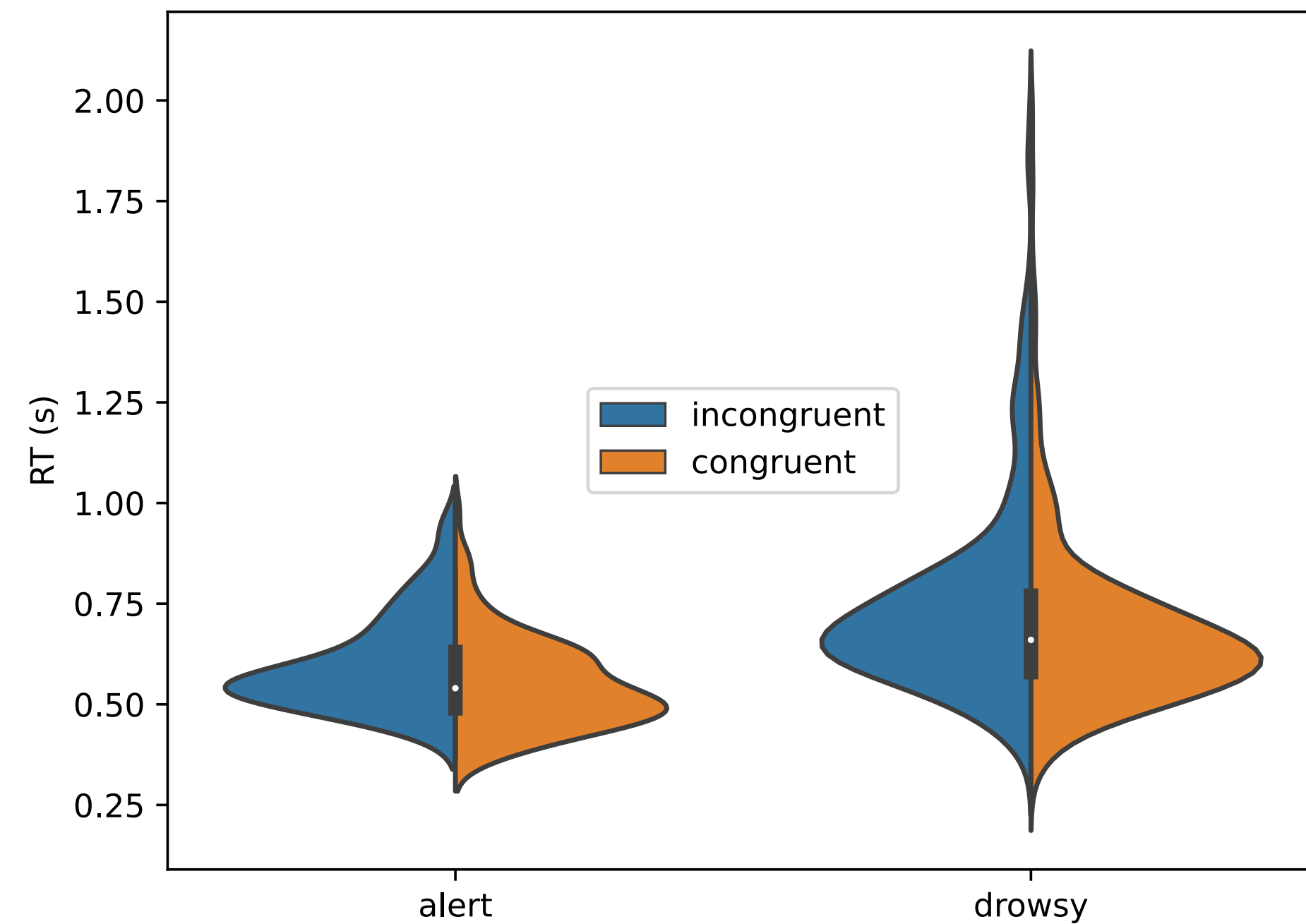
96% HPDI: [0.68 0.94]

50% HPDI: [0.79 0.88]



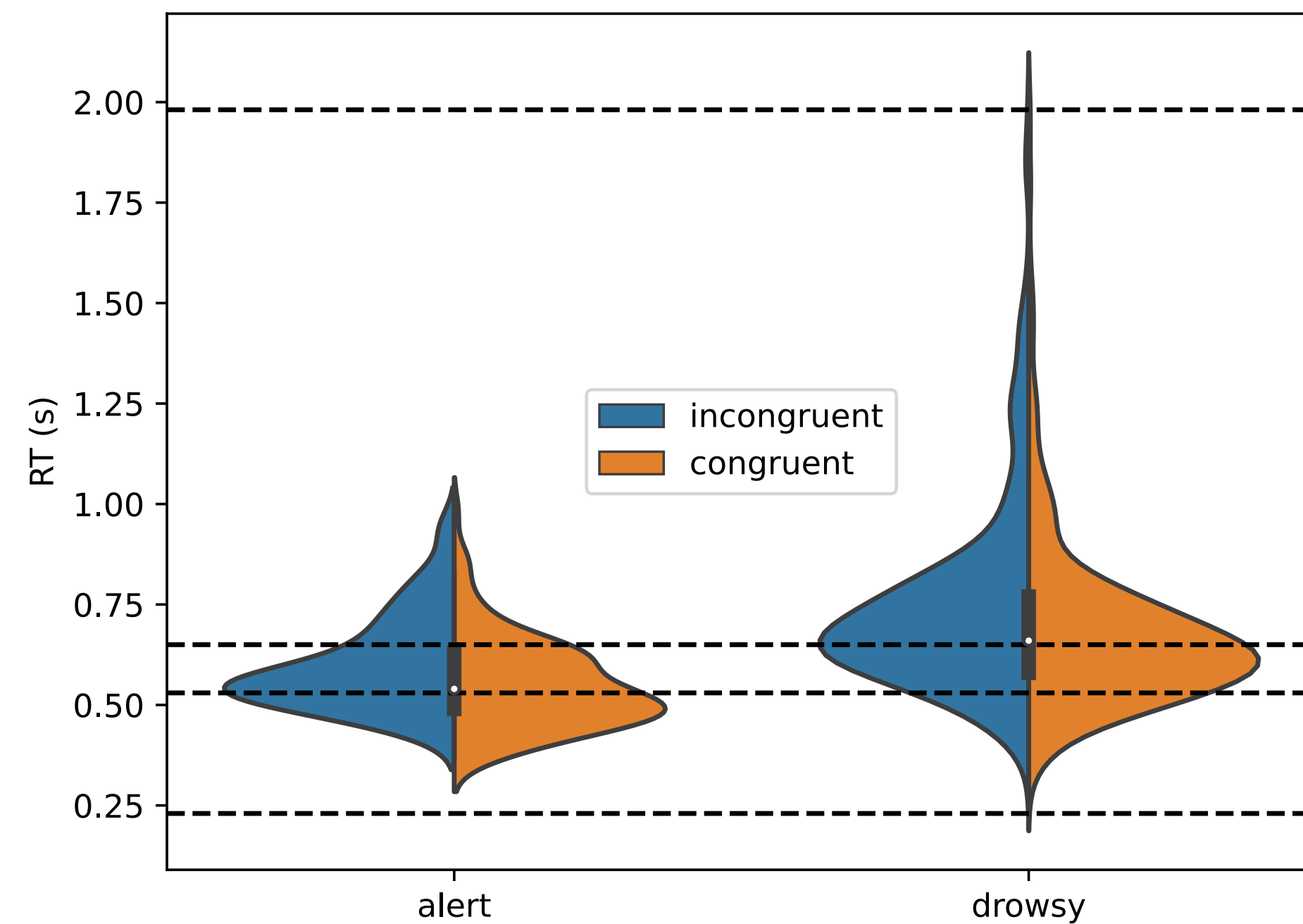
Information Theory

- Non-parametric, robust, within-participant statistical tests
- Bin RT within-participant: top 3rd, middle 3rd, bottom 3rd of RT



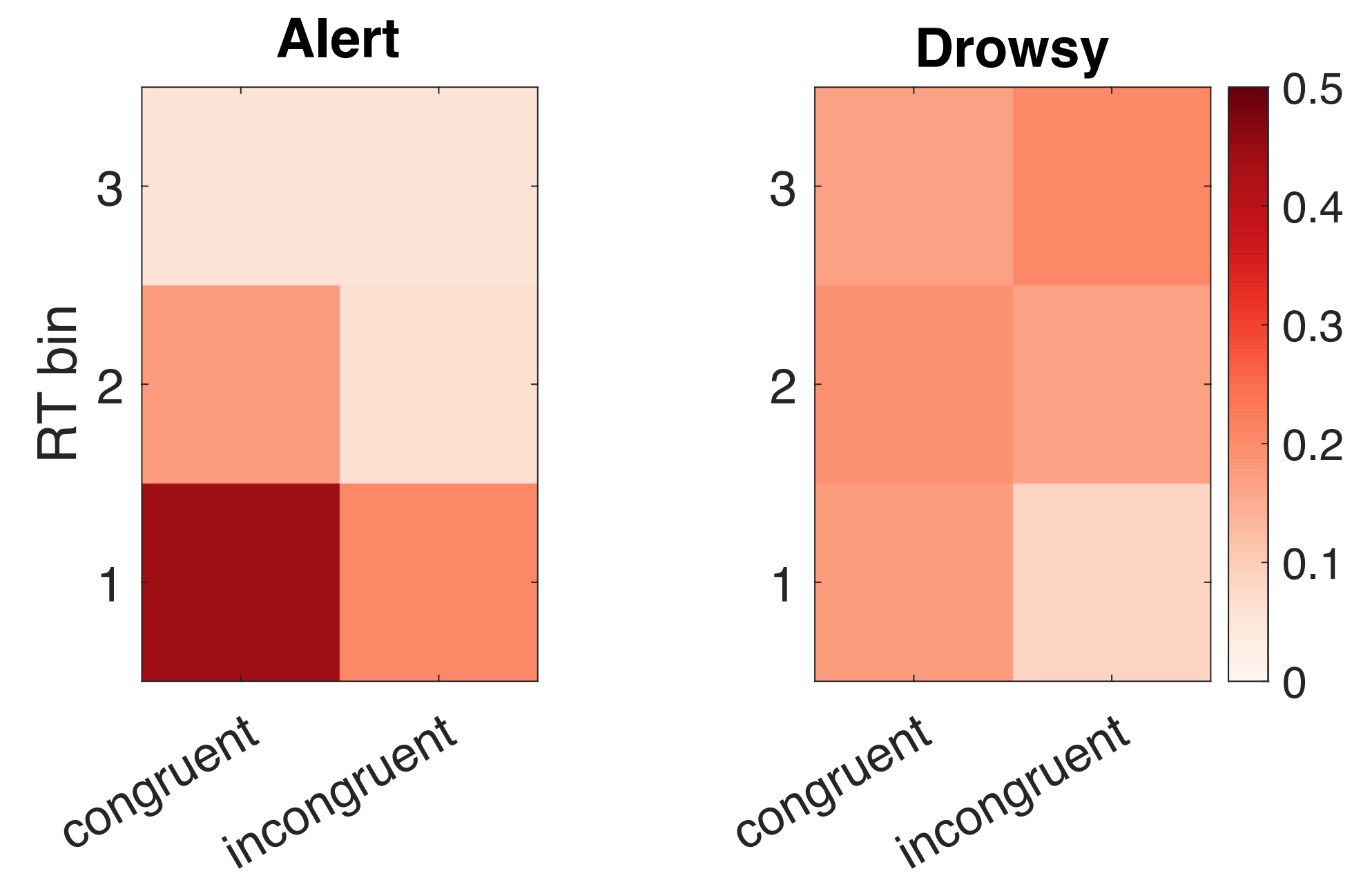
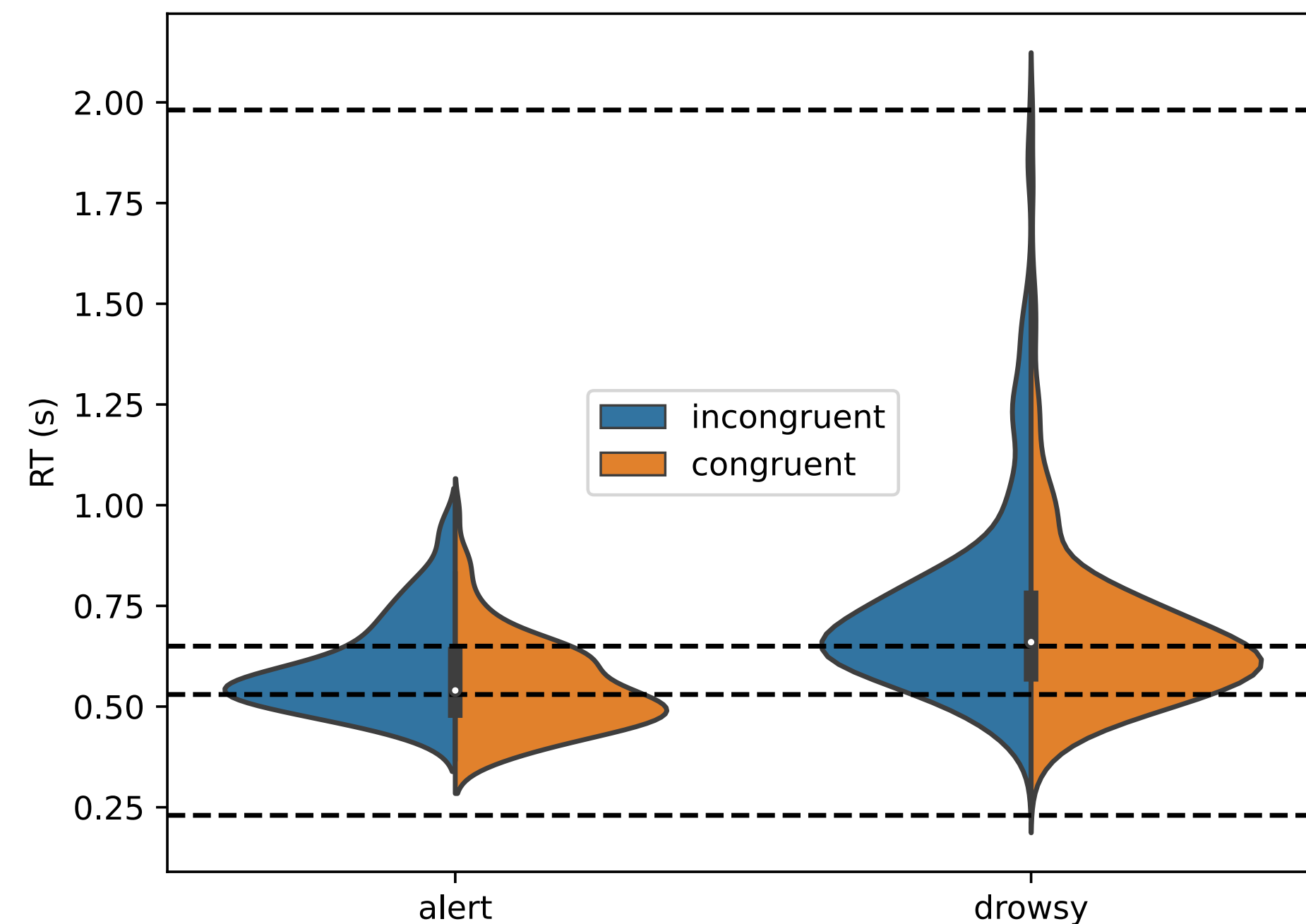
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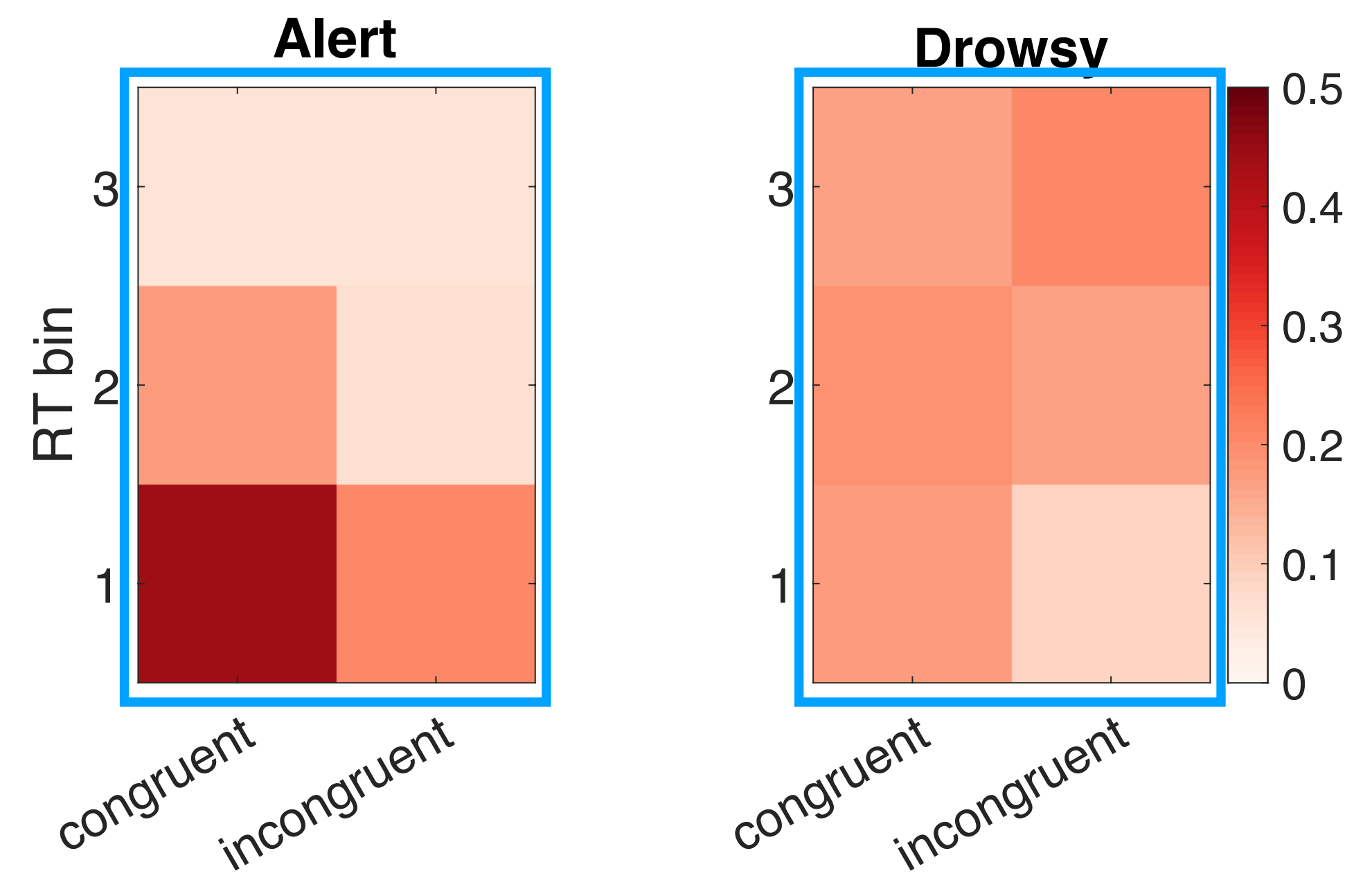
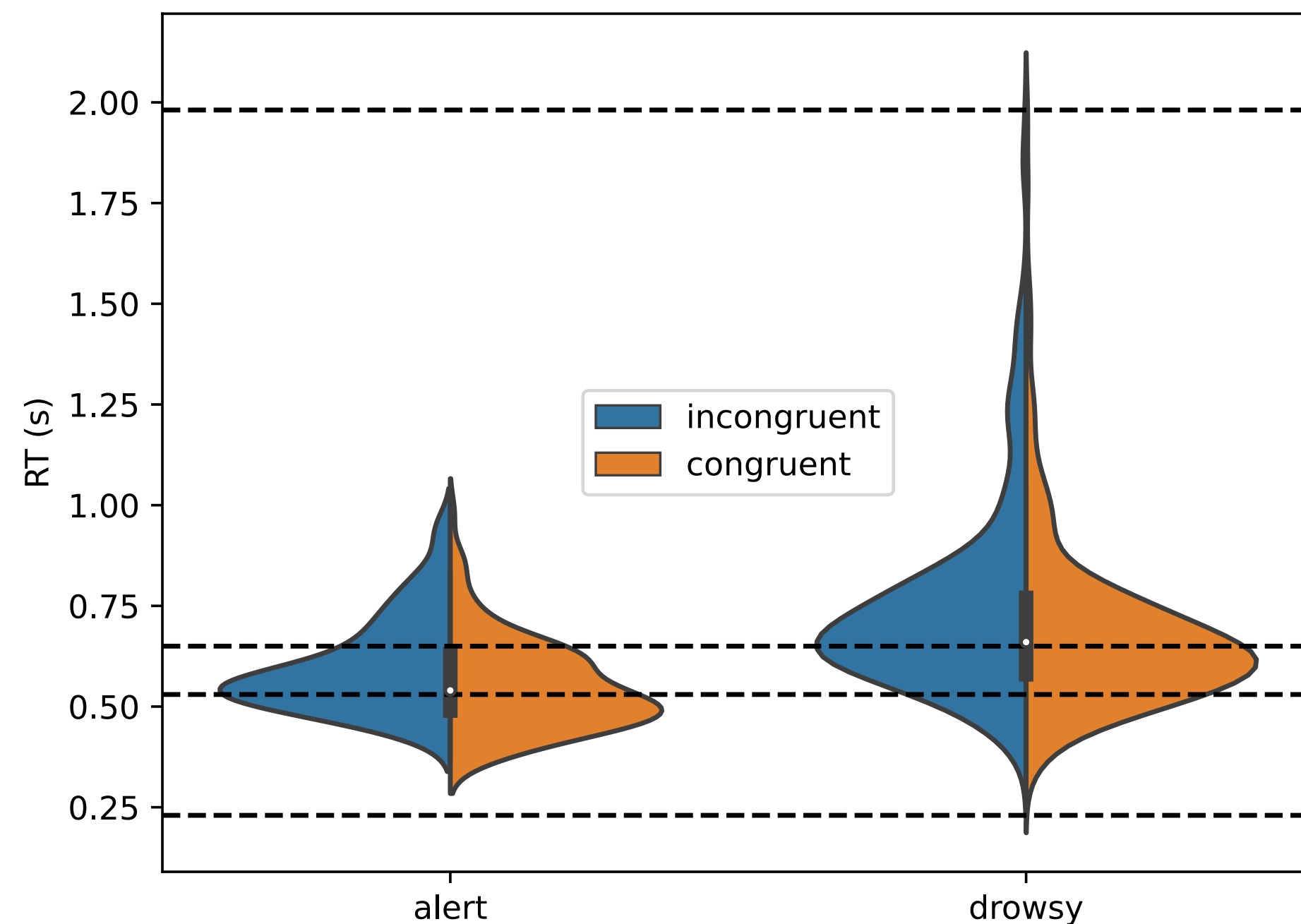
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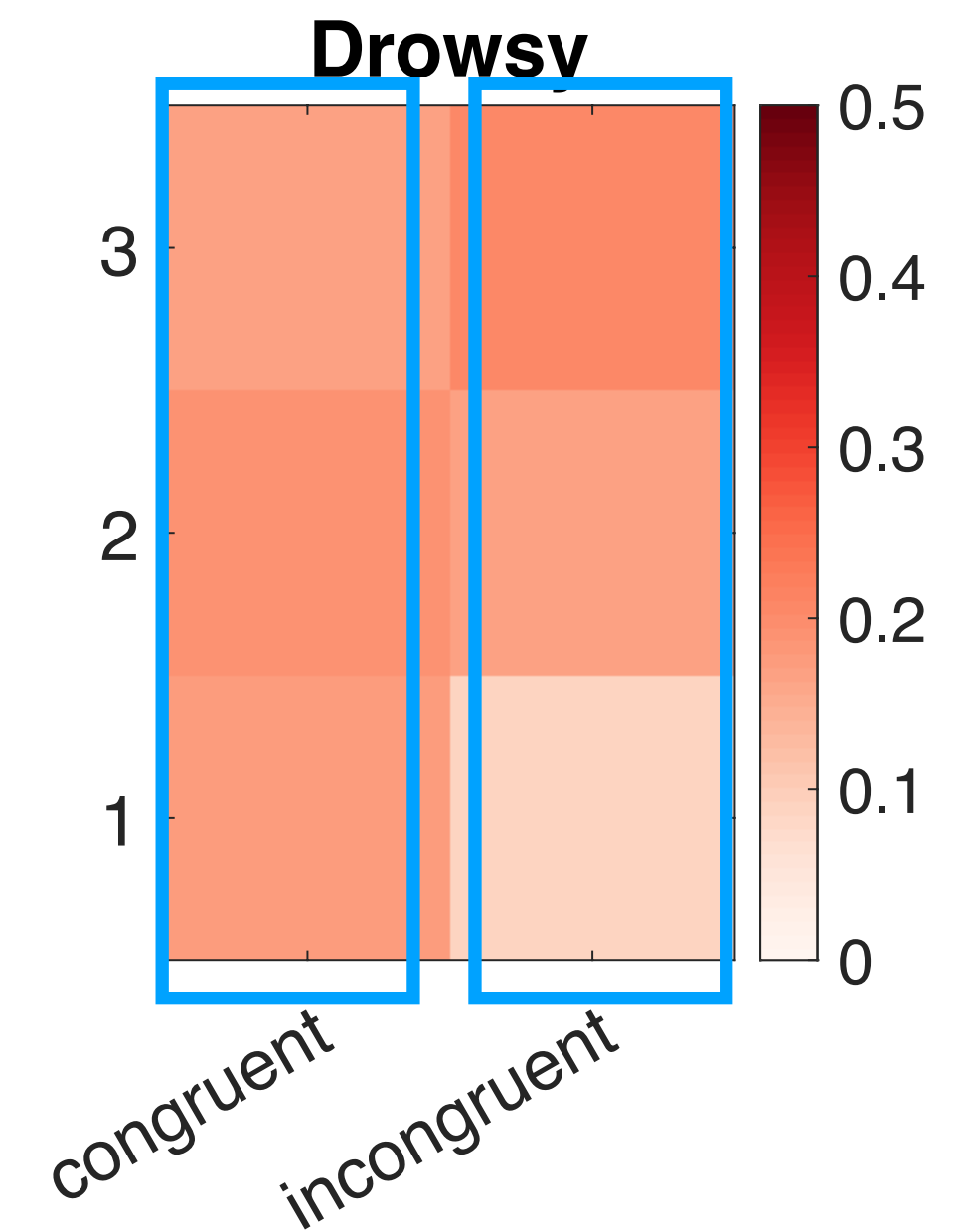
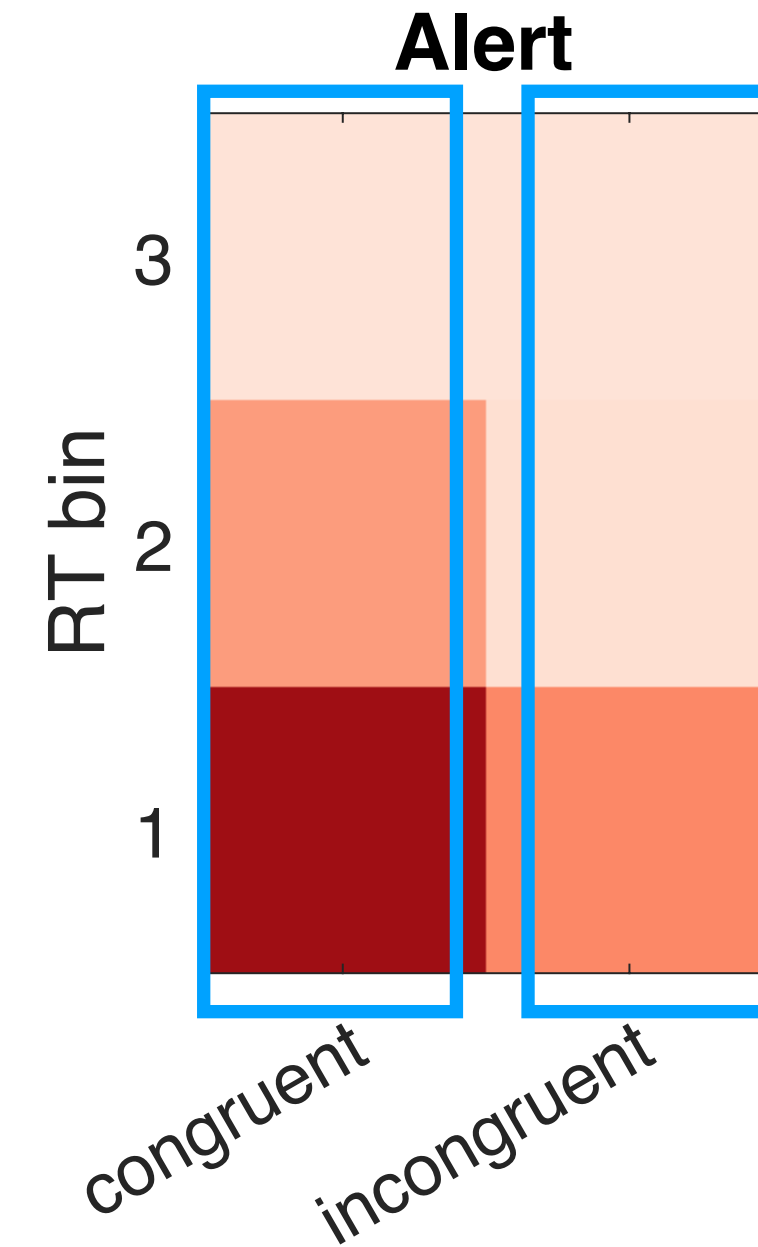
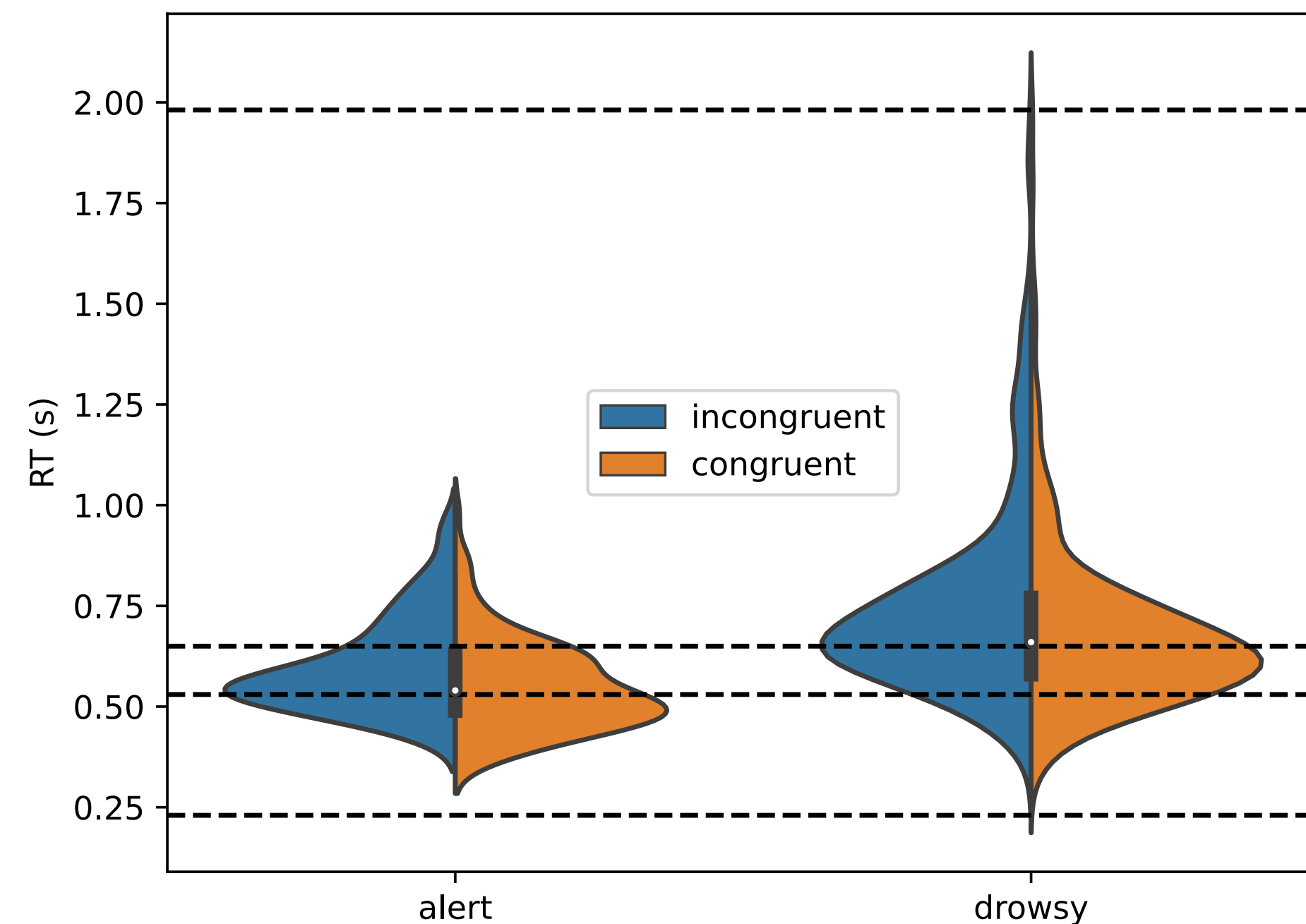
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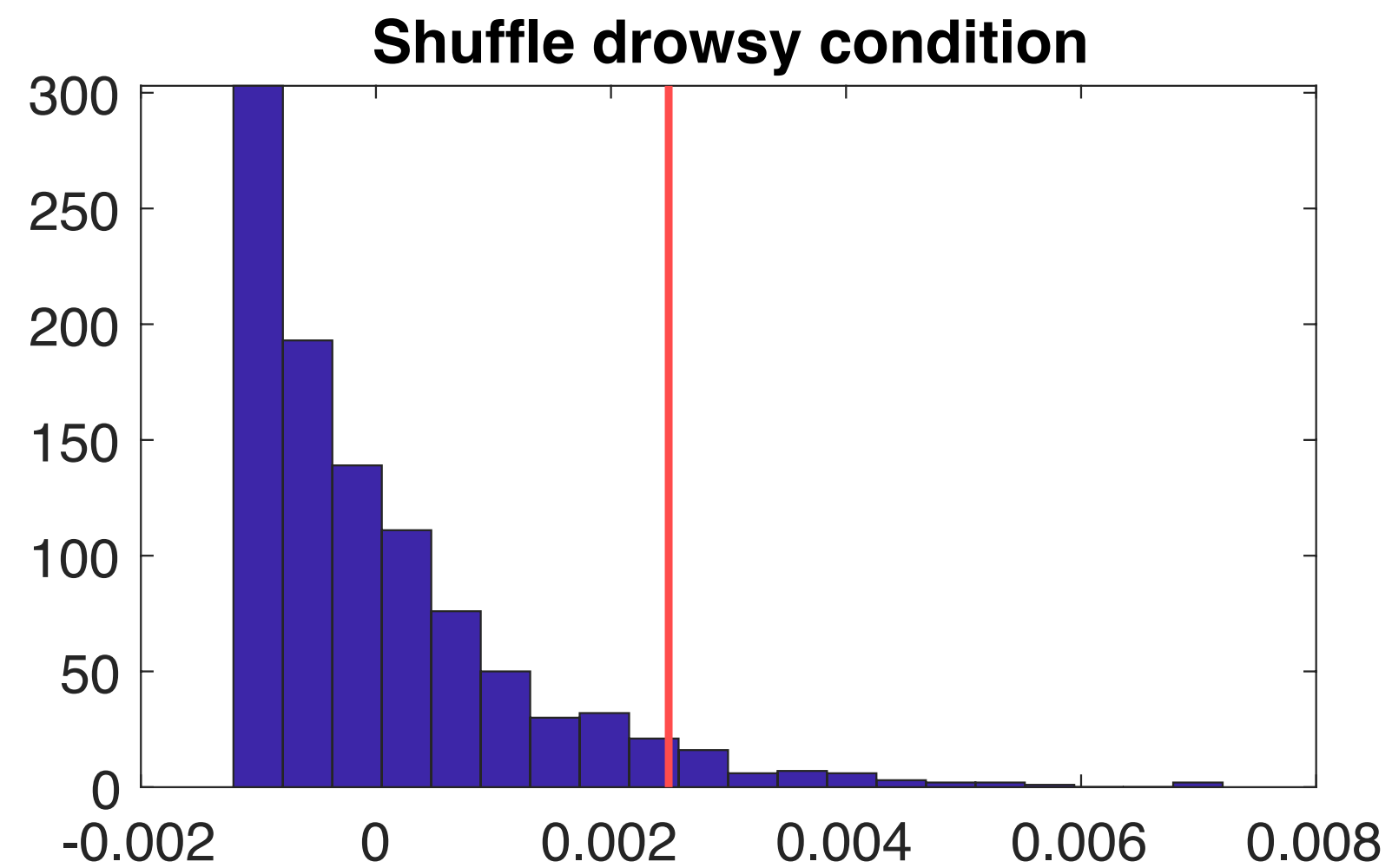
Information Theory

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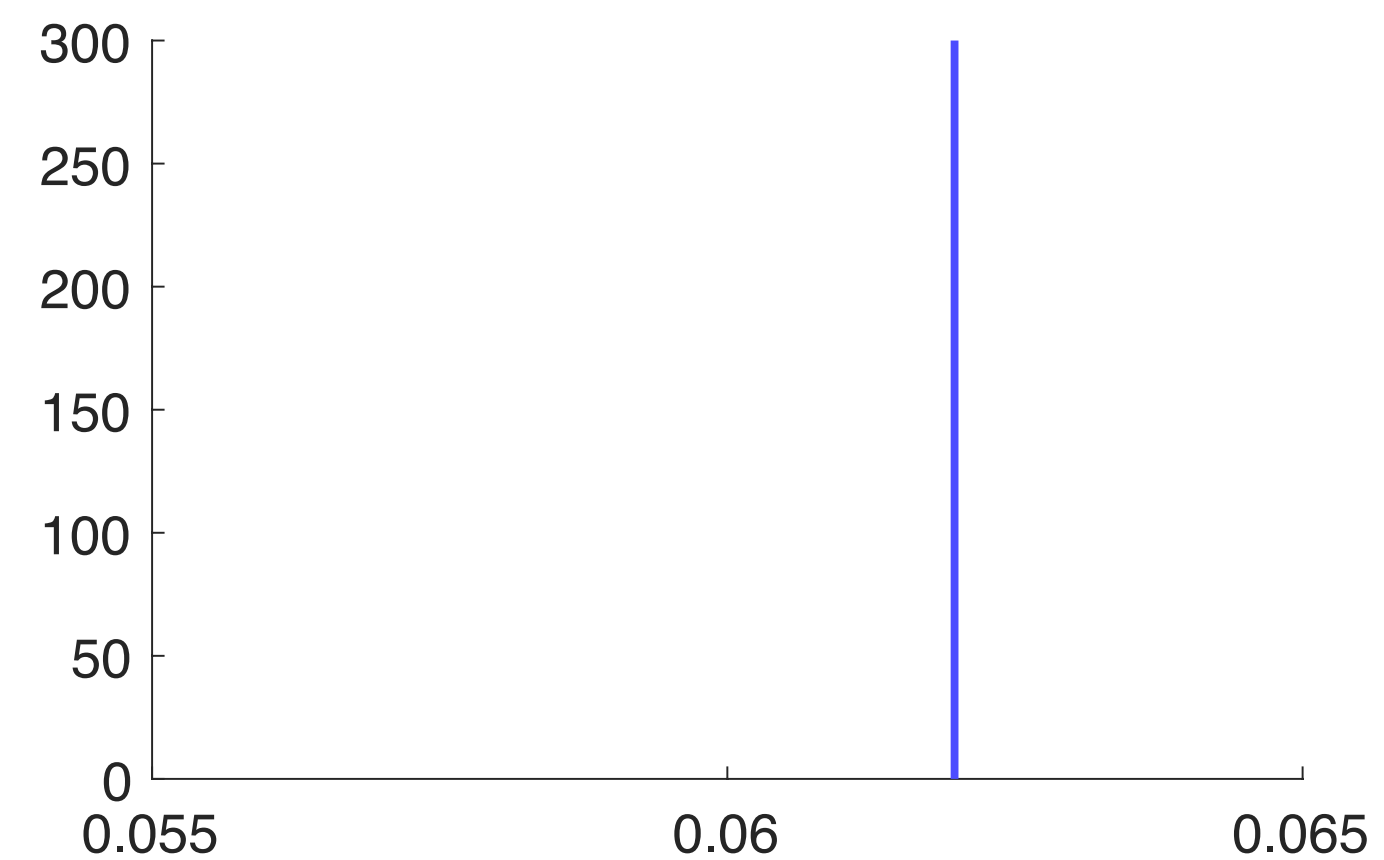


Non-parametric permutation test

- Only assumes independence of trials
- Shuffle **drowsy** condition label



...



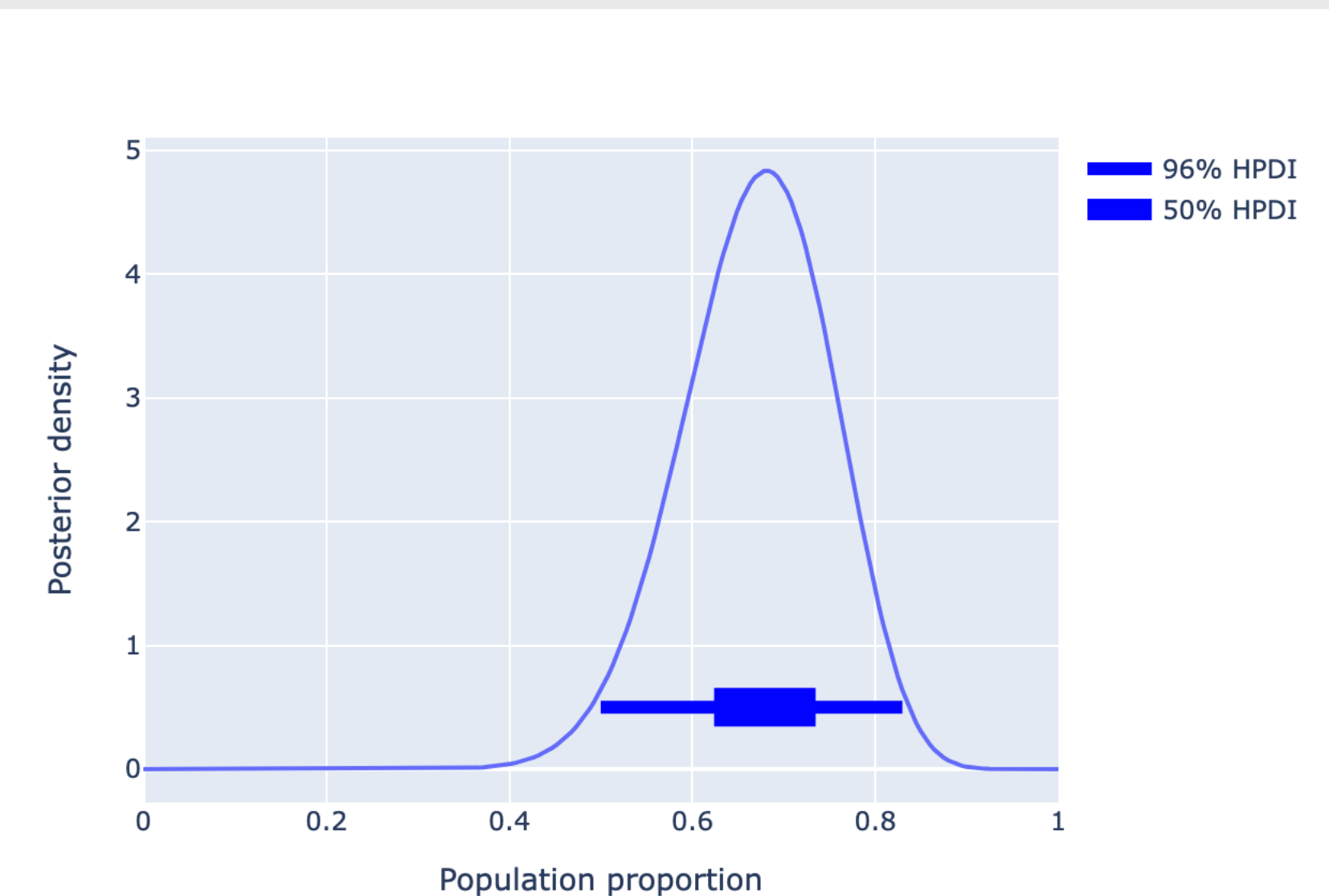
Within-participant MI

- **Congruence** : MI(congruence; RT) : 23/33
Can detect congruence effect in majority of population with this experiment
(LM: 5/33 or 11/33 with logRT)

Prevalence MAP: 0.68

96% HPDI: [0.50 0.83]

50% HPDI: [0.62 0.73]

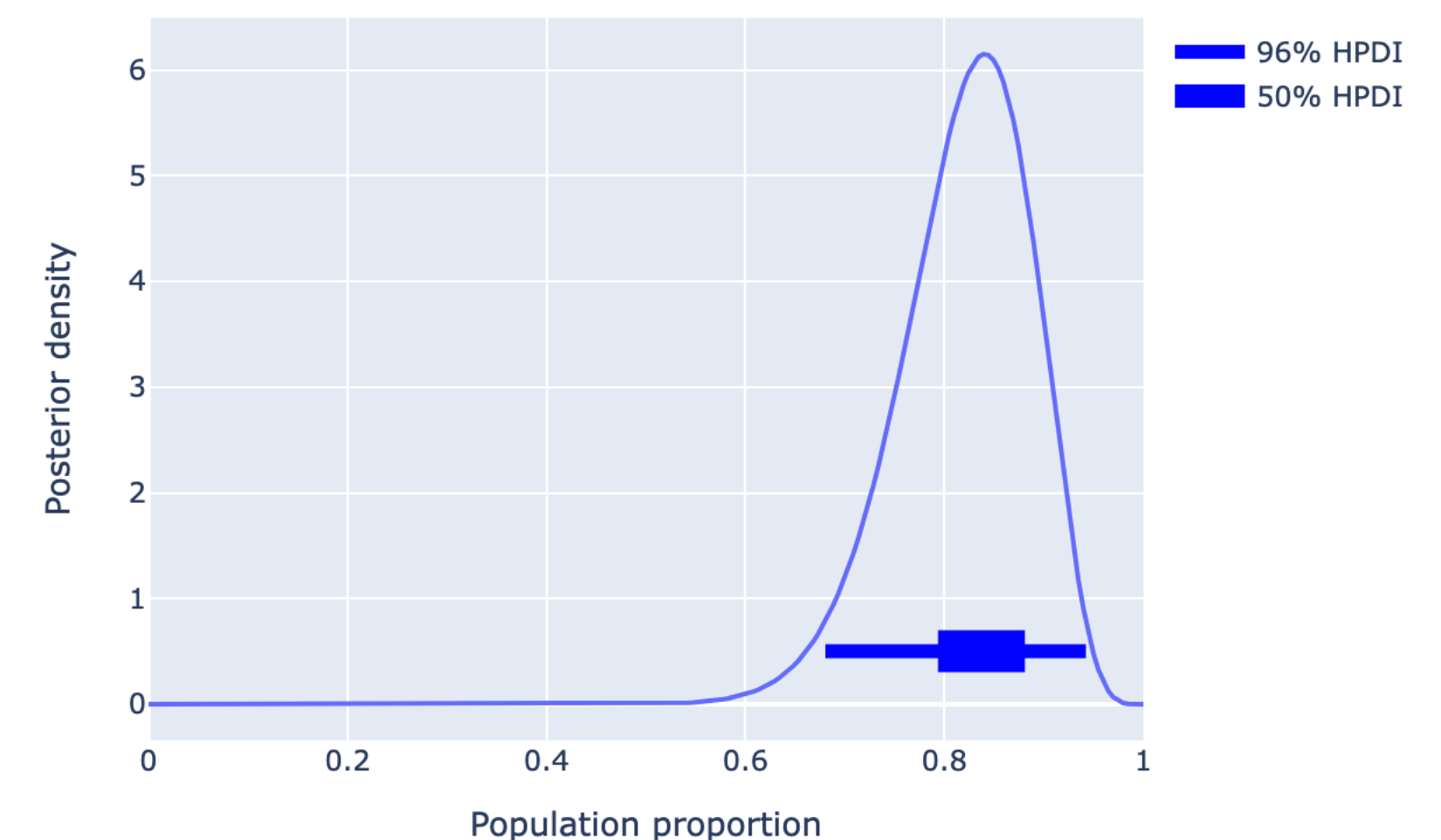


Within-participant MI

- **Congruence** : MI(congruence; RT) : 23/33
Can detect congruence effect in majority of population with this experiment
(LM: 5/33 or 11/33 with logRT)
- **Alertness**: MI(alertness; RT) : 28/33
(LM: 28/33 or 29/33 with logRT)

Prevalence MAP: 0.68
96% HPDI: [0.50 0.83]
50% HPDI: [0.62 0.73]

Prevalence MAP: 0.84
96% HPDI: [0.68 0.94]
50% HPDI: [0.79 0.88]



Within-participant Interaction

- co-Information: $\text{col}(\text{RT}; \text{congruence}; \text{alertness})$
= $\text{MI}(\text{RT}; \text{congruence}) - \text{CMI}(\text{RT}; \text{congruence} \mid \text{alertness})$
- **modulation index** for effect of **alertness** on the *relationship* between **RT** and **congruence**
- permutations: shuffle alertness (null distribution that alertness has no effect)

Within-participant Interaction

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= $\text{MI}(\text{RT}; \text{congruence}) - \text{CMI}(\text{RT}; \text{congruence} \mid \text{alertness})$
- **modulation index** for effect of **alertness** on the *relationship* between **RT** and **congruence**
- permutations: shuffle alertness (null distribution that alertness has no effect)
- 22/33 have a significant interaction!

Prevalence MAP: 0.65

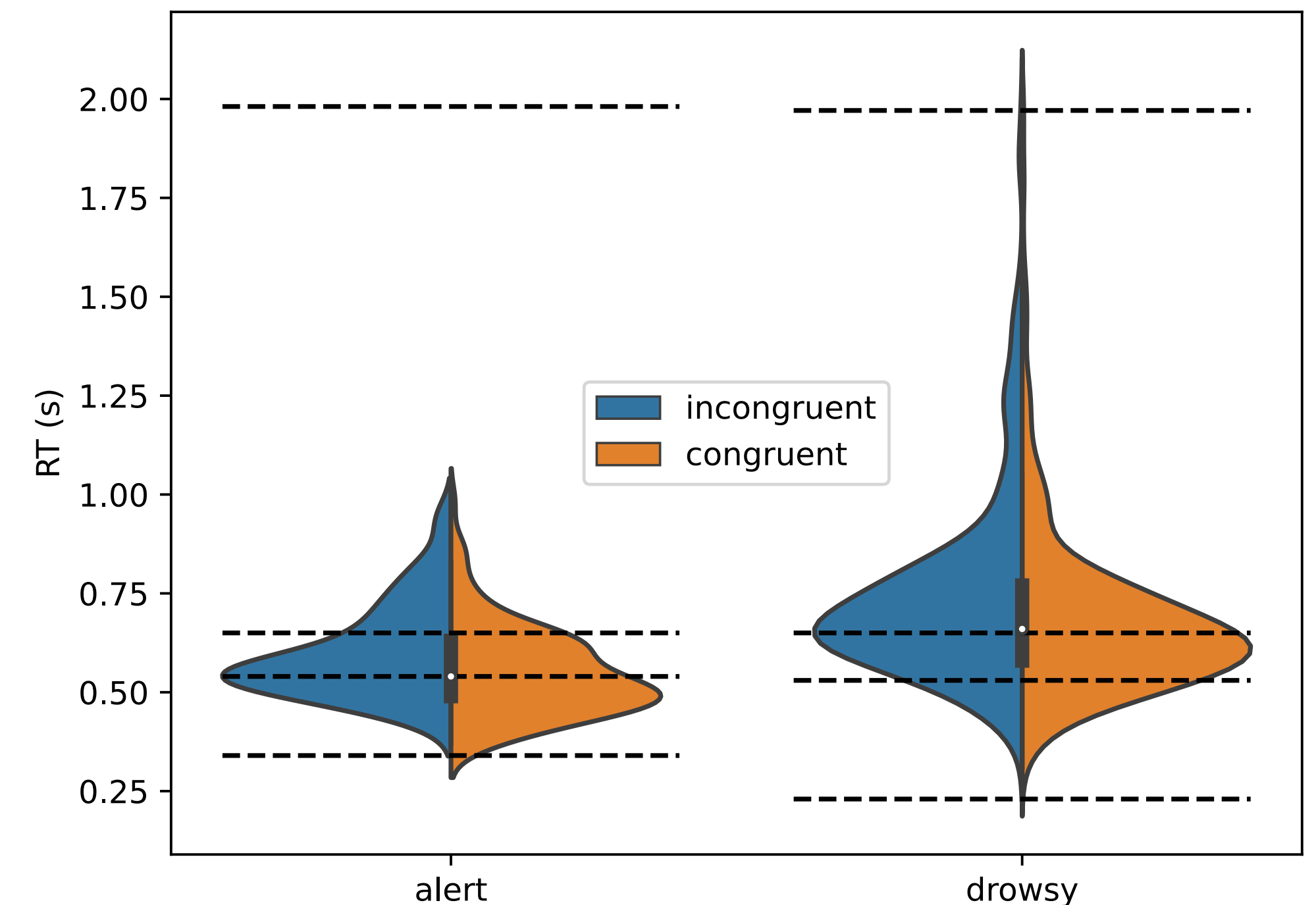
96% HPDI: [0.47 0.80]

50% HPDI: [0.59 0.70]

Within-participant interaction

Normalising within alertness condition

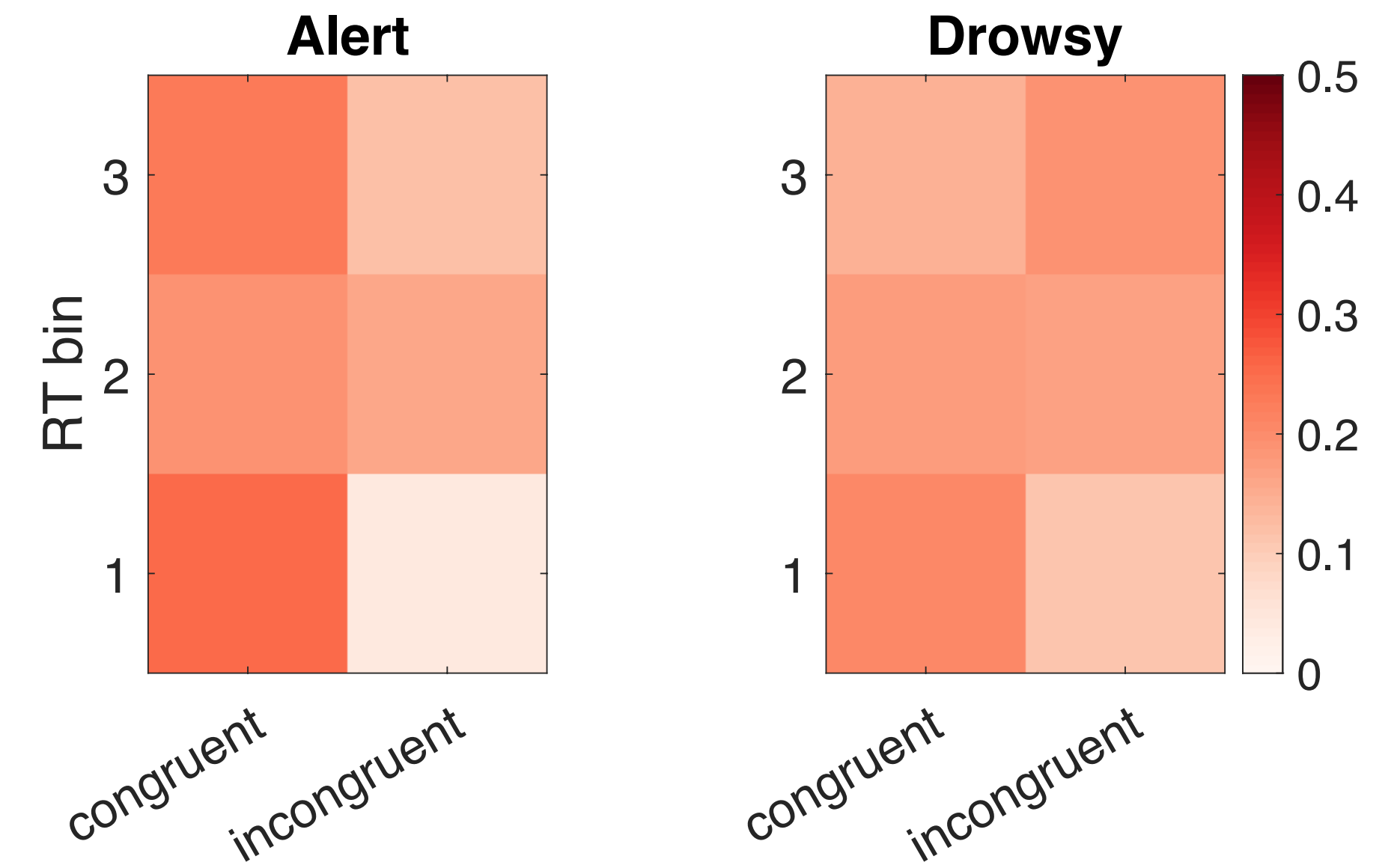
- There is a strong effect of drowsiness. Could this interaction be due to different rank ordering across the conditions because of the fixed effect of drowsiness?
- Can bin RT in each condition separately. This is like z-scoring in each condition: both now have same uniform marginal distribution.



Within-participant interaction

Normalising within alertness condition

- No difference in marginal distribution between drowsy and alert. Effect of congruence visible in both conditions
- **Alertness:** $MI(\text{alertness}; RT) : 0/33$
(we have normalised away effect of alertness)
- **Congruence :** $MI(\text{congruence}; RT) : 26/33$
- **Interaction:** $col(RT; \text{congruence}; \text{alertness}) : 25/33$



Within-participant interaction

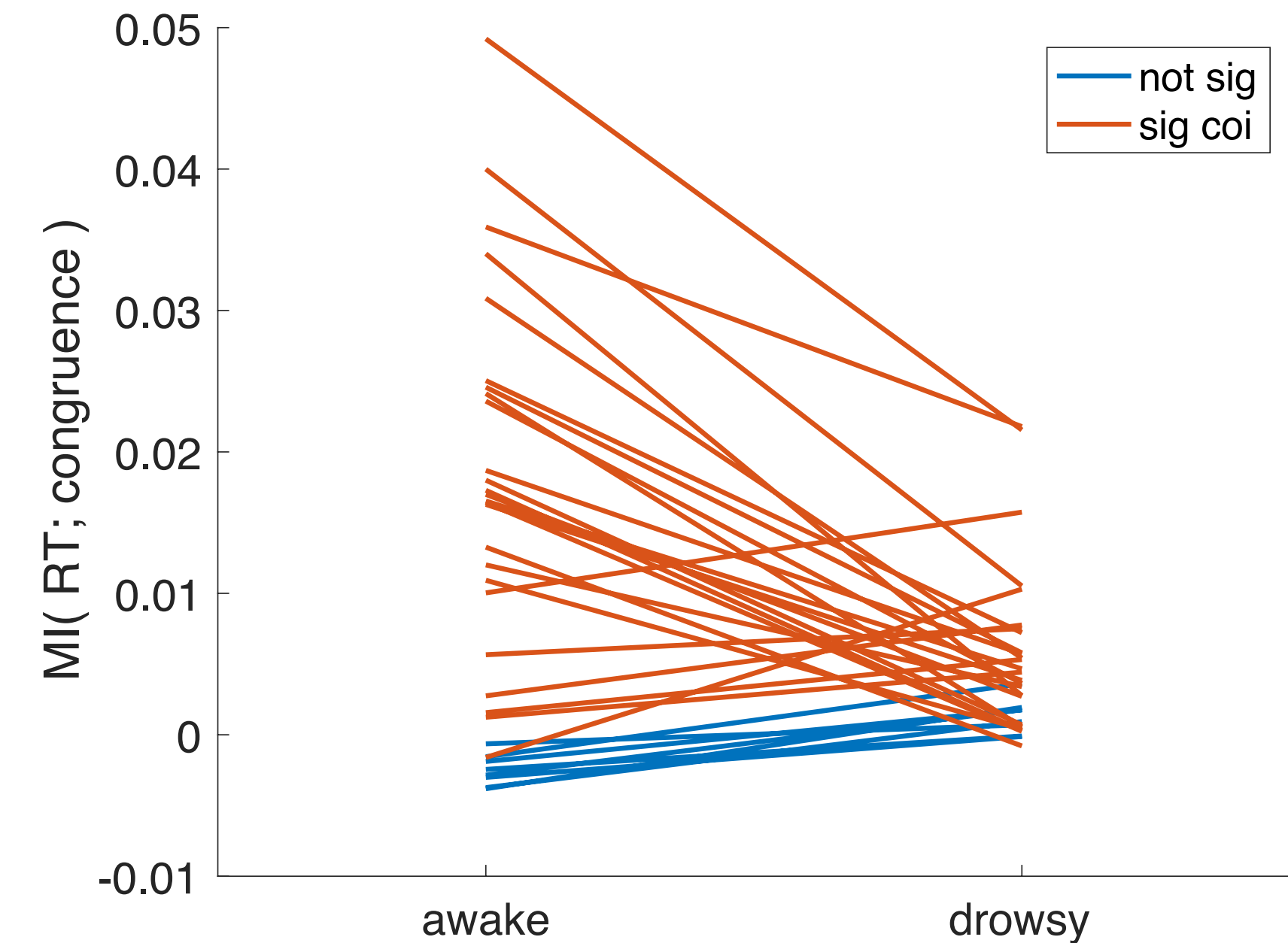
Normalising within alertness condition

- **Interaction:** col(RT; congruence; alertness) : 25 /33
- Interaction remains even when normalising away marginal differences between alertness conditions.
- It is a difference in the mapping between congruence and RT rank, within each condition

Within-participant interaction

Normalising within alertness condition

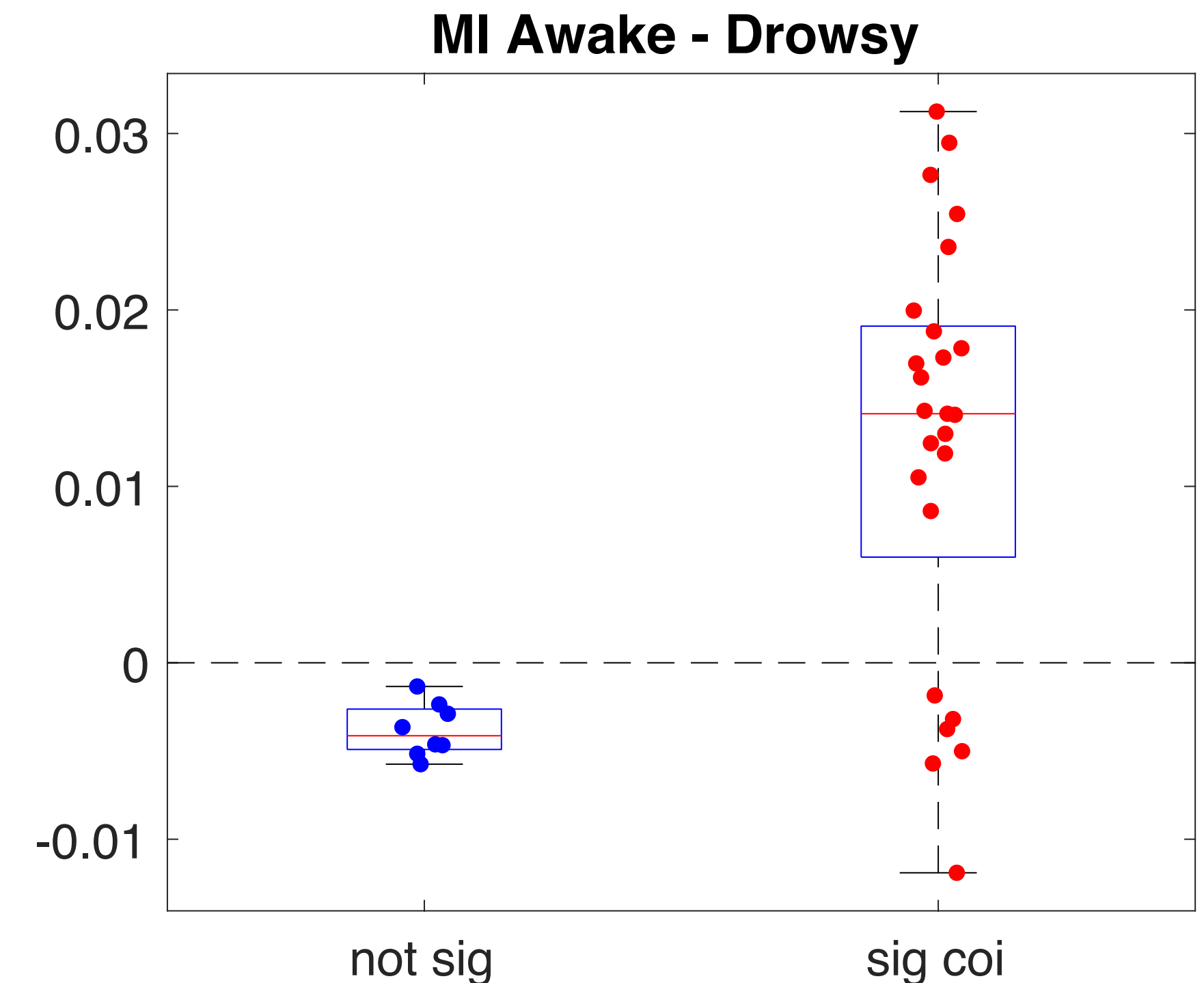
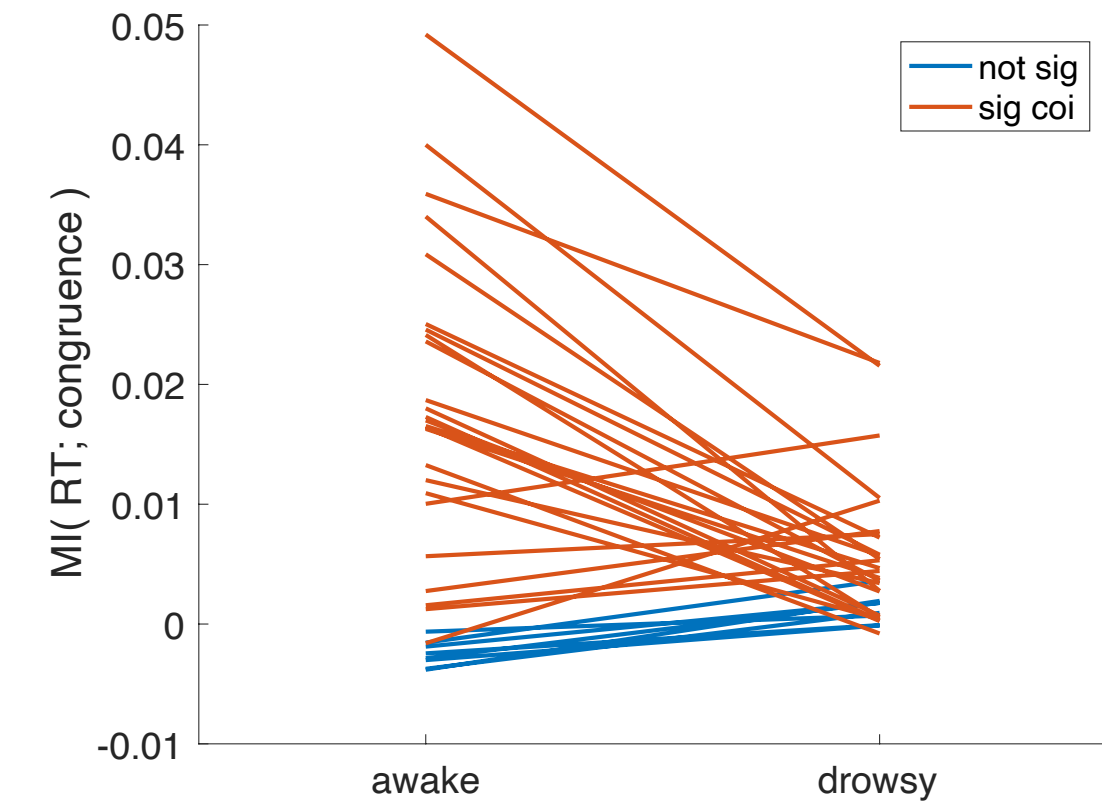
- **Interaction:** col(RT; congruence; alertness) : 25 /33
- Interaction remains even when normalising away marginal differences between alertness conditions.
- It is a difference in the mapping between congruence and RT rank, within each condition
- Majority of participants with sig. col have weaker effect of congruence on RT in the drowsy condition



Within-participant interaction

Normalising within alertness condition

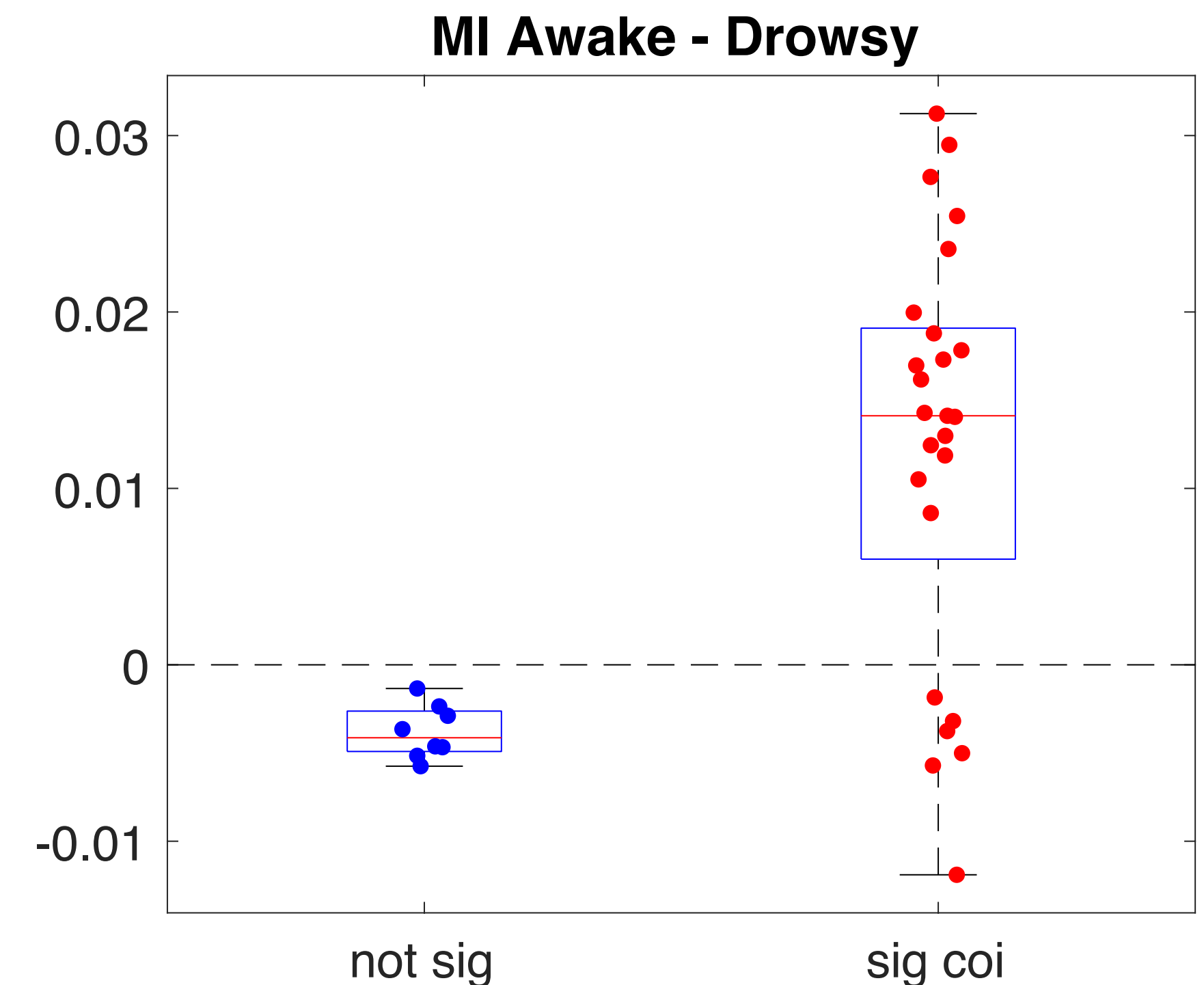
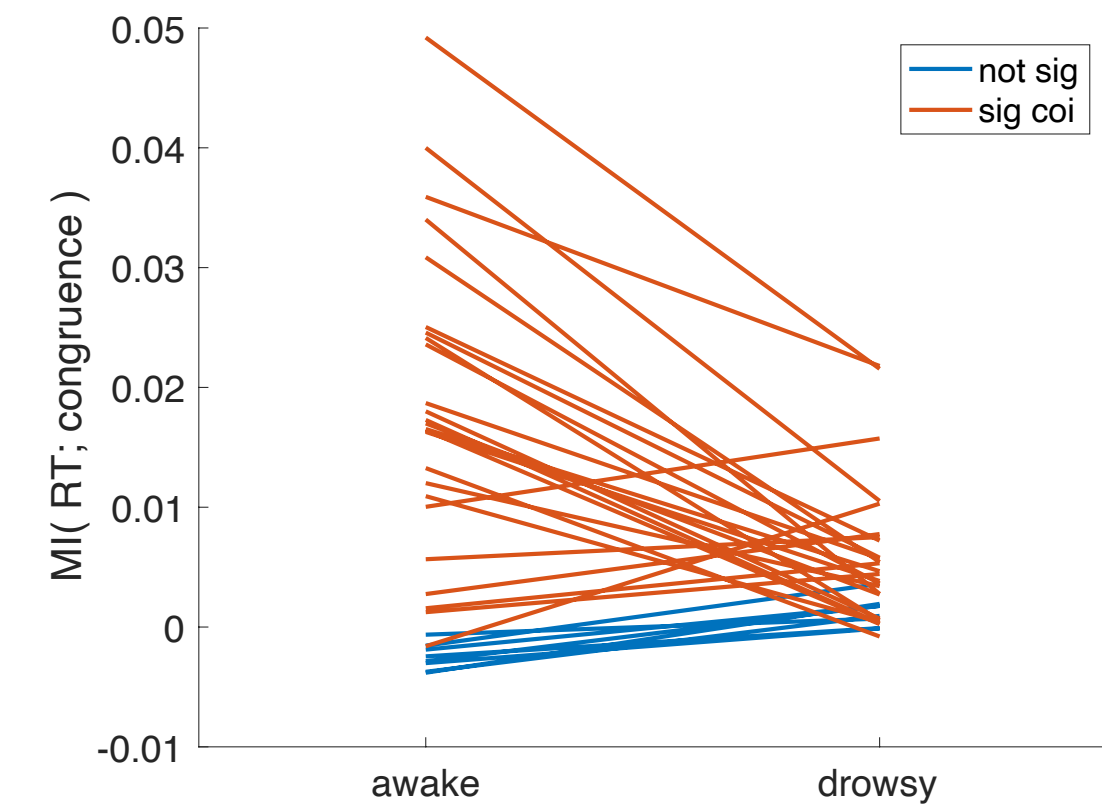
- **Interaction:** col(RT; congruence; alertness) : 25 /33
- Majority of participants with sig. col have lower MI(RT; congruence) in the drowsy condition
- This means: can better predict congruence condition from observing reaction times when they are awake
- I.e. 0.05 bits : need to observe 20 trials on average to predict congruence
0.02 bits : need to observe 50 trials to predict congruence



Within-participant interaction

Normalising within alertness condition

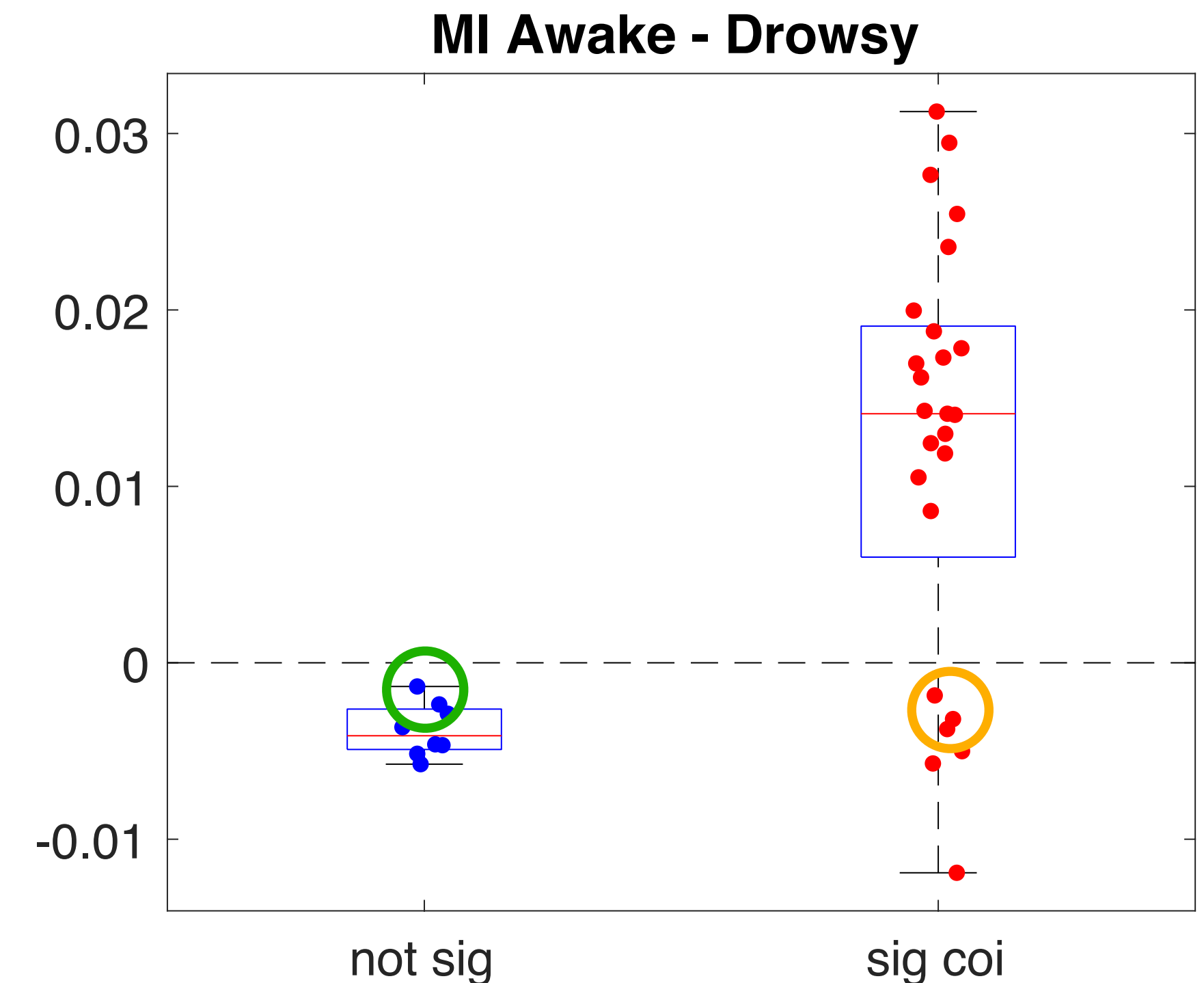
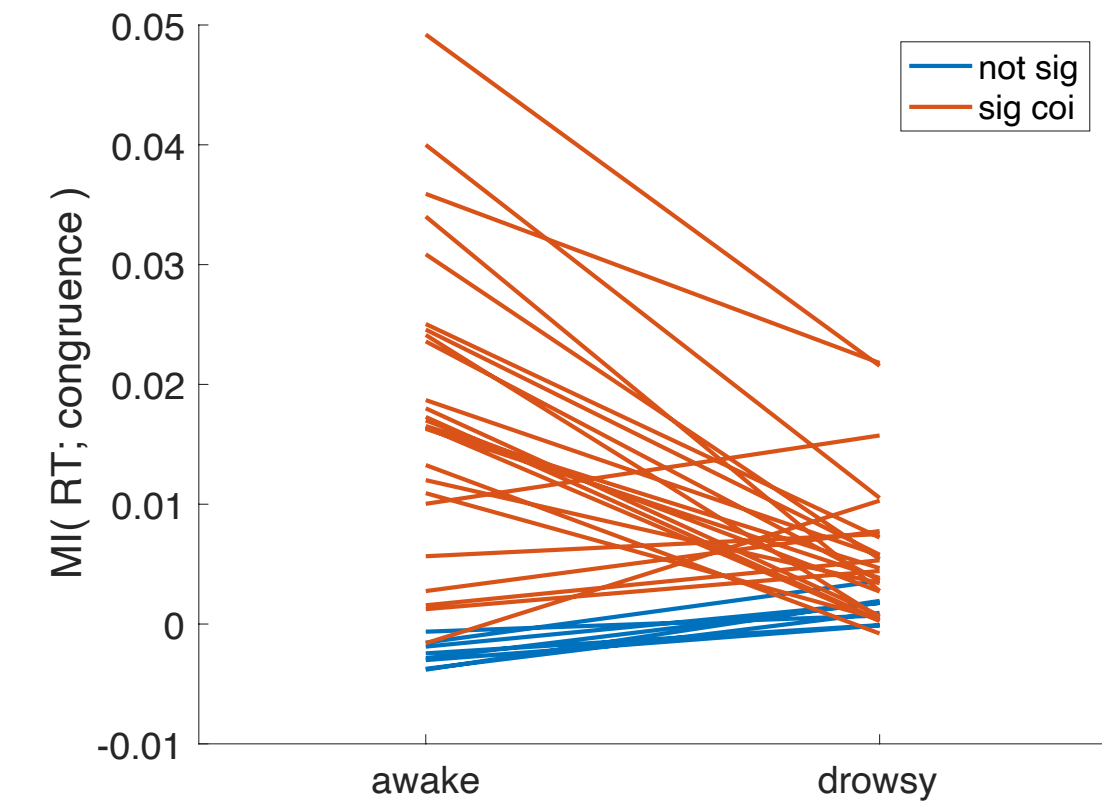
- **Interaction:** col(RT; congruence; alertness) : 25 /33
- Majority of participants with sig. col have lower MI(RT; congruence) in the drowsy condition
- But not all!
One has more MI in drowsy condition, and another group have differences the same range as participants without an interaction
- So what is col measuring here?



Within-participant interaction

Normalising within alertness condition

- **Interaction:** col(RT; congruence; alertness) : 25 /33
- Majority of participants with sig. col have lower MI(RT; congruence) in the drowsy condition
- But not all!
One has more MI in drowsy condition, and another group have differences the same range as participants without an interaction
- So what is col measuring here?

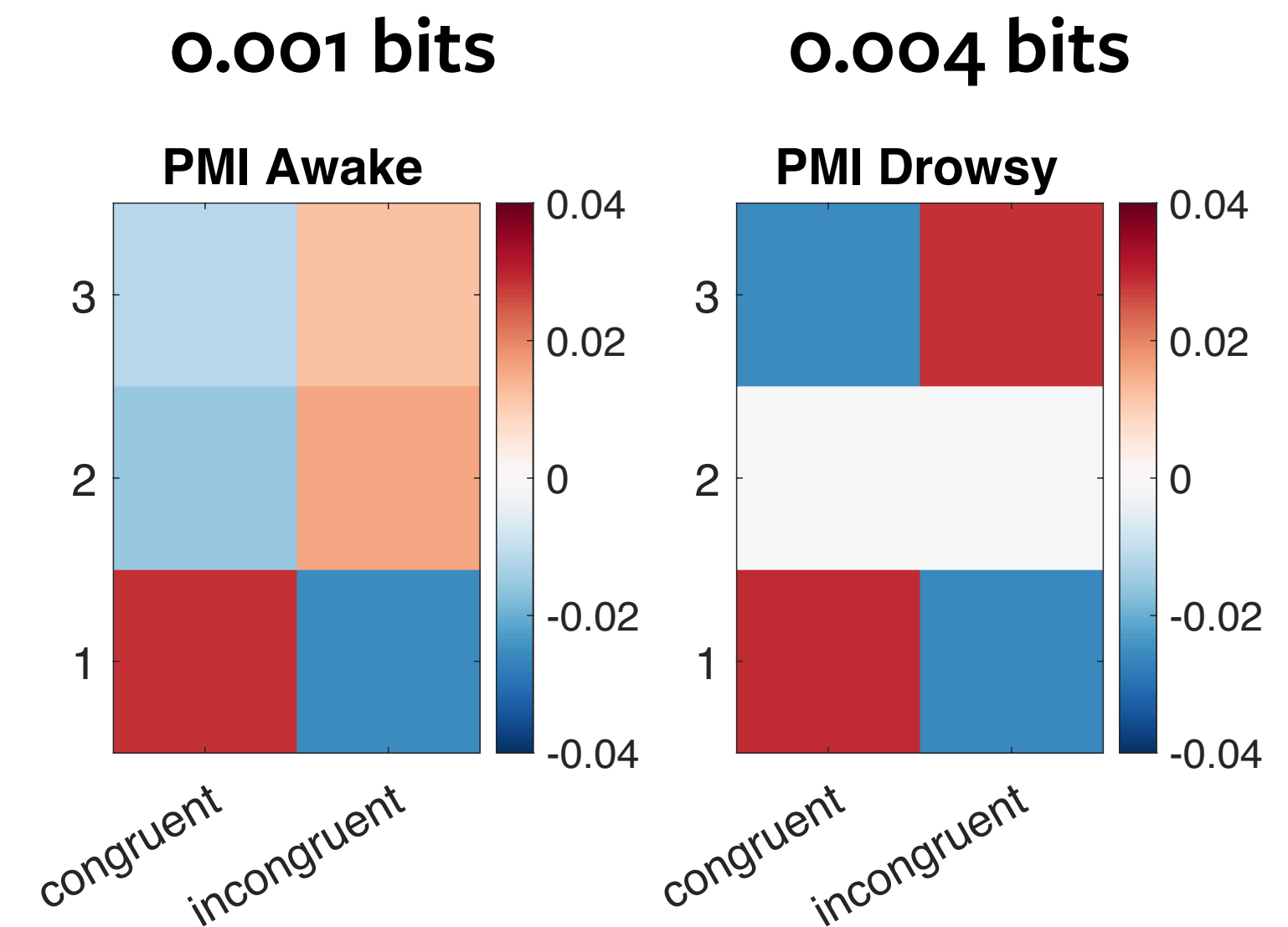
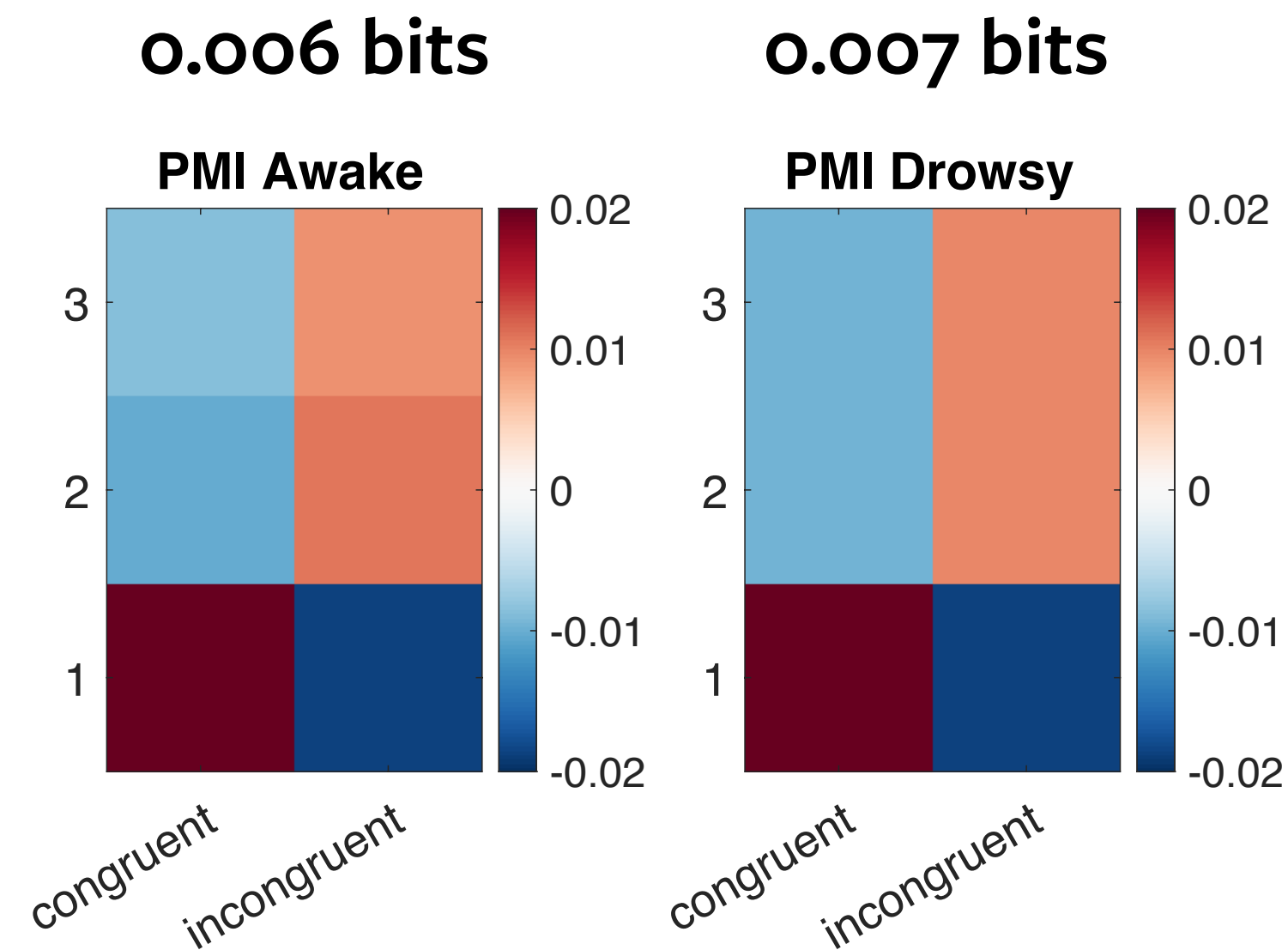


Within-participant Interaction

What is col measuring for those participants with no MI difference?

- Breakdown MI(RT; congruence) into the contributions of each combination : pointwise mutual information (PMI)

$I(RT; congruence)$



Population mean theta power

Linear mixed-effects model fit by ML

Model information:

Number of observations	31494
Fixed effects coefficients	4
Random effects coefficients	132
Covariance parameters	11

Formula:

RT ~ 1 + congruent*drowsy + (1 + congruent*drowsy | ID)

Model fit statistics:

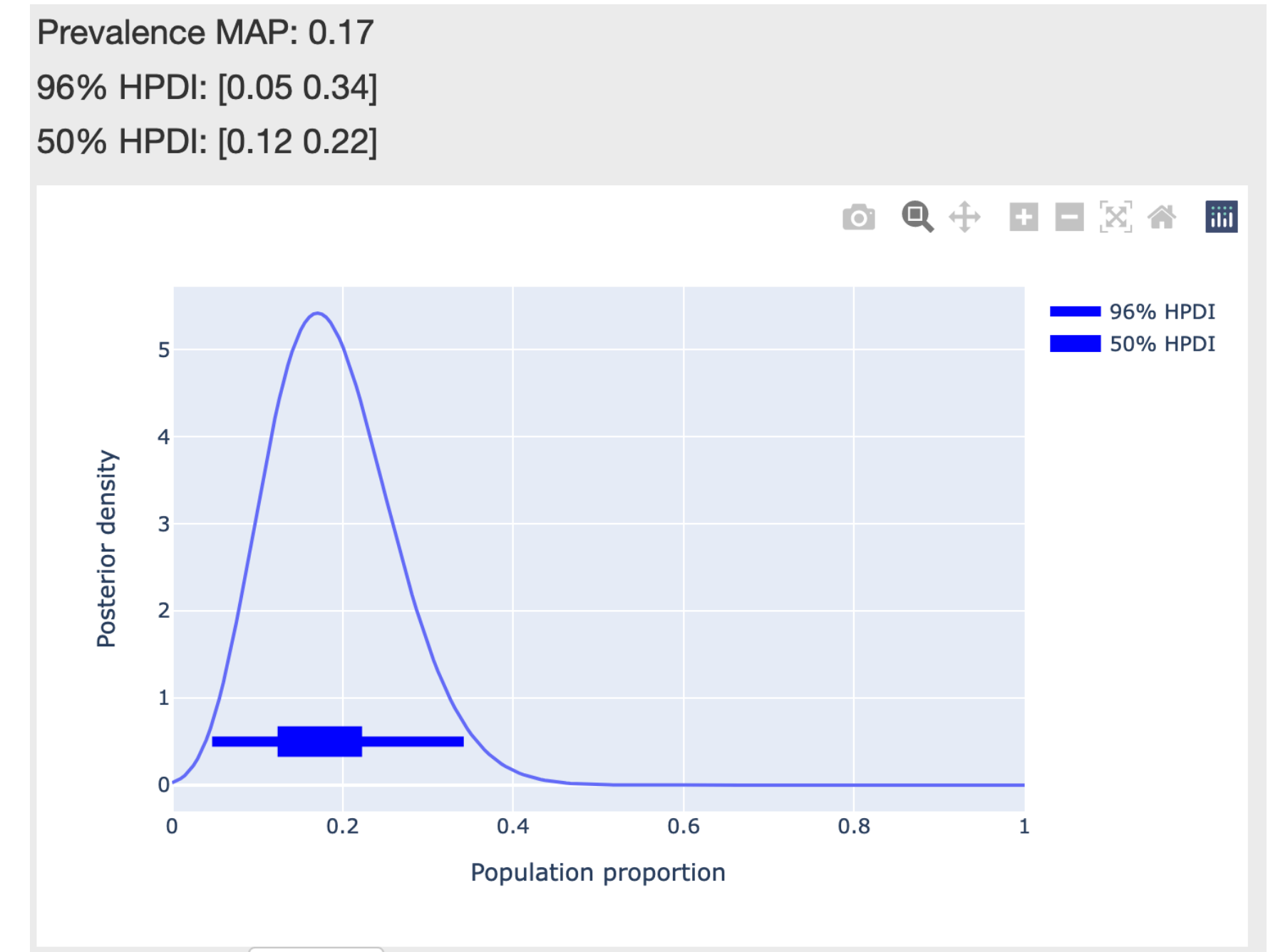
AIC	BIC	LogLikelihood	Deviance
186002.103418761	186127.466703719	-92986.0517093807	185972.103418761

Fixed effects coefficients (95% CIs):

Name	Estimate	SE	tStat	DF	pValue
{ '(Intercept)' }	0.1731843078659	0.188766712004545	0.91745152536073	31490	0.358913109686981
{ 'congruent' }	0.48460173439805	0.106257660414205	4.56062868793664	31490	5.11926886598408e-06
{ 'drowsy' }	-0.711831267208987	0.120697539366188	-5.89764522911559	31490	3.72484753626096e-09
{ 'congruent:drowsy' }	-0.360403111762771	0.125389902393751	-2.87425944898679	31490	0.0040524718335375

Within-participant theta power

- Linear Model: congruence: 6/33, alertness: 13/33, interaction: 5/33
- MI: congruence: 6/33, alertness: 18/33, co-information: 7/33
- MI approach has slightly more sensitivity within-participant but shows similar effects.
- 7/33 provides evidence of effect at population level
- Supports group level interaction, but suggests either experiment underpowered for within participant, or not everyone shows it



Within-participant theta power

Prevalence MAP: 0.17

96% HPDI: [0.05 0.34]

50% HPDI: [0.12 0.22]

Fixed effects coefficients (95% CIs):

Name	Estimate	SE	tStat	DF	pValue
{ '(Intercept)' }	0.1731843078659	0.188766712004545	0.91745152536073	31490	0.358913109686981
{ 'congruent' }	0.48460173439805	0.106257660414205	4.56062868793664	31490	5.11926886598408e-06
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{ 'congruent:drowsy' }	-0.360403111762771	0.125389902393751	-2.87425944898679	31490	0.0040524718335375

Random effects covariance parameters (95% CIs):

Group: ID (33 Levels)

Name1	Name2	Type	Estimate
{ 'congruent:drowsy' }	{ 'congruent:drowsy' }	{ 'std' }	0.391678112624746

What does col tell us?

- There is a statistical interaction between the alertness condition and the congruence effect
- This is more general / less specific than an interaction in a linear model for means
- Can do non-parametric statistical testing within-participant with higher power than linear modelling
- Prevalence lets us make quantitative statements about the population, based on our experiment, which may be closer to the scientific aims
- More robust statistical results (replication built in across participants)

Advantages of information theory

- Computationally efficient within-participant non-parametric tests with good statistical power
- col can reflect different types of interaction
- Can tell you where something interesting is happening, but not exactly what it is (i.e. no mechanistic insight here!)
- But can help with multiple comparison (selecting regions for modelling), exploratory data analysis, prevalence etc.

If you want to estimate prevalence online just go to:

<https://estimate.prevalence.online/>

LMEM on logRT

```
Linear mixed-effects model fit by ML

Model information:
  Number of observations      54264
  Fixed effects coefficients      4
  Random effects coefficients  132
  Covariance parameters      11

Formula:
  logRT ~ 1 + congruent*drowsy + (1 + congruent*drowsy | ID)

Model fit statistics:
  AIC      BIC      LogLikelihood      Deviance
  33466    33600    -16718              33436

Fixed effects coefficients (95% CIs):
  Name              Estimate      SE      tStat      DF      pValue      Lower
  {'(Intercept)'}    6.4152    0.033187    193.3    54260      0      6.3501
  {'congruent'}       0.050629  0.0056099    9.025    54260    1.854e-19    0.039634
  {'drowsy'}          0.24012   0.034498    6.9604    54260    3.4319e-12    0.1725
  {'congruent:drowsy'} -0.0017643 0.0067334   -0.26202    54260    0.79331    -0.014962
```